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AN EXPERIMENTAL STUDY OF TWO METHODS OF LONG DIVISION

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K. G. F.

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AN EXPERIMENTAL STUDY OF TWO METHODS OF LONG DIVISION

CHAPTER I

INTRODUCTION

Long division is considered by many teachers one of the most difficult topics in the elementary school program in arithmetic. It is sometimes suggested that this topic be deferred to a later position in the curriculum with the expectation that more mature pupils will find it easier to learn. Difficulty with long division, however, is not limited to elementary school pupils. It is not unusual for an adult to be confused or inaccurate when performing long division. The author has found a substantial number of college students seriously deficient in this process of arithmetic. Hence the problem of long division does not seem to be entirely one of the proper grade placement of instruction. Nor does it seem to be entirely a problem of computational facility. The difficulties of some individuals appear to be rooted in a lack of understanding of the long division process itself.

When an adult performs an arithmetical process, such as addition or multiplication, he usually follows the same procedure regardless of the size or kind of the numbers involved. In the case of long division, however, his procedure is not likely to be uniform. Although he may have been taught a systematic plan for performing this process, he often has forgotten or discarded it and proceeds by using various devices that he has developed for himself. It is the trial and error demanded in the long division process to obtain the correct quotient figures that seems to trouble elementary school pupils and adults alike.

It was the purpose of this study to investigate the possibilities of using in the elementary school a method of long division in which the first step is the construction of a table of multiples of the divisor. If, when performing long division, the first nine multiples of the divisor are available, then the quotient figures can be obtained by inspection instead of by trial and error, and the required products of the divisor and the quotient figures need not be computed, for they can be taken directly from the table. Therefore the method of long division under trial in this investigation appeared to have sufficient advantages to justify its study. It seemed also that a table of multiples of the divisor might be of assistance in developing an understanding of the long division process.

Before making a formal comparison of this experimental method of long division with the method now in general use it was necessary to develop, from the experiences of an informal preliminary study, methods of teaching the experimental method to elementary school pupils.

THE USUAL METHOD OF LONG DIVISION

The method of performing long division usually taught in contemporary elementary schools requires that the figure in the quotient be estimated one way or another and that an estimated quotient figure be tried out and corrected, if necessary, before proceeding with the division. When this usual method is employed, an estimated quotient figure must often be corrected, and sometimes corrected as many as five times.

A frequently taught form of the usual method of performing long division will be illustrated by dividing 181345 by 26. In the first step 181 is selected as the partial dividend. To estimate the number of times the divisor is contained in the partial dividend, 18 is divided by 2, the first figure of the divisor. This gives 9 for a trial quotient figure. The product of 26 and 9 is greater than 181, hence 9 is rejected and 8 is tried out as the required quotient figure. Since the product of 26 and 8 is also greater than 181, 8 is rejected and 7 is tried. This figure must also be rejected, and then 6 is tried and accepted as the correct quotient figure. The quotient figure is now placed over the last figure of the partial dividend. The product, 156, of the divisor and the

$$\begin{array}{r@{}l}
 & 6974 \\
26) & \overline{181345} \\
 & \underline{156} \\
 & 253 \\
 & \underline{234} \\
 & 194 \\
 & \underline{182} \\
 & 125 \\
 & \underline{104} \\
 & 21
\end{array}$$

quotient figure is placed under the partial dividend and subtracted. The remainder must be less than the divisor; therefore it is compared with the divisor before continuing. Finally, the next digit in the dividend is brought down to form the second partial dividend of 253. In estimating the next quotient figure 25 is divided by 2, giving 12. The largest possible quotient figure is 9, hence 9 is used for the trial quotient figure in this case. It is tried and accepted as the second quotient figure. In determining the third and fourth quotient figures two corrections of the estimated quotient figure are needed in each case.

It is granted that an experienced computer would be able to eliminate some of the trials in solving the preceding example. Furthermore, it is possible to construct division examples which would require fewer changes of the estimated quotient figures. However, Upton has shown that this procedure of using the first figure of a 2-figure divisor as a trial divisor gives the correct quotient figure on the first trial in only 66.70 per cent of all cases.[1] Hence the necessity of correcting an estimated quotient figure is by no means unusual.

The following form of the usual method of long division is sometimes used. If the second figure of a 2-figure divisor is 5 or less, the first figure of the divisor is used as the trial divisor; but if the second figure of the divisor is greater than 5, the first figure of the divisor is increased by 1 before it is used as the trial divisor. For example, if the divisor is 53 the trial divisor is 5, but if the divisor is 58 the trial divisor is 6.

When Upton studied all the partial dividends that can possibly occur for each 2-figure divisor he found that this plan gave the correct quotient figure on the first trial in 80.43 per cent of all cases, a quotient figure requiring one correction in 19.29 per cent of all cases, and a quotient figure requiring two corrections in .28 per cent of all cases.[2] In no situation was it necessary to make more than two corrections. This is therefore the form of the usual method which seems to offer the promise of best results in the elementary school, and it was selected for use in this study for comparison with the experimental method.

---

[1] Clifford B. Upton, "Making Long Division Automatic," *Tenth Yearbook, National Council of Teachers of Mathematics,* p. 277, 1935.

[2] *Ibid.,* p. 275.

Either of these two forms of the usual method of performing long division offers abundant opportunity for confusion and error, especially for the beginner. The estimation, testing, and correction of quotient figures, the required multiplication and subtraction, and the added difficulties encountered when there are zeros in the quotient and when there is a remainder, make long division a hard process for the child in the elementary school to learn and to perform.

That many pupils fail to achieve a high degree of competence in long division with the usual method was indicated by the results obtained when 520 fifth grade pupils in a number of elementary schools were given Test III, which will be found on page 16, at the end of the school year. These pupils had studied

TABLE 1

TEST III SCORES OF 520 FIFTH GRADE PUPILS

No. of Examples Correct	*No. of Pupils*
5	91
4	111
3	110
2	83
1	67
0	58

TABLE 2

TIME USED FOR TEST III BY 520 FIFTH GRADE PUPILS

No. of Minutes	*No. of Pupils*
31–35	271
26–30	71
21–25	70
16–20	51
11–15	48
6–10	9
1–5	

long division under average conditions. Their intelligence quotients ranged from 65 to 142 with a mean of 98.4. Their scores on this test in long division are shown in Table 1. Of these pupils, only 18 per cent solved all five of the test examples correctly and only 39 per cent solved at least four examples correctly. The time used by this group for Test III is given in Table 2. No instructions in regard to checking were given the pupils, but an examination of the test papers showed that about one-third of the pupils took the time to check their work. Of the 58 pupils making a score of zero, 39 pupils worked 35 minutes, which was the maximum time allowed.

The Experimental Method of Long Division

The division example on page 2 will now be solved by using the method of long division under consideration in this study. The first step in this experimental method of long division is to construct and check a table of the multiples of the divisor by using only addition and multiplication by 2. Each even multiple is obtained by doubling a value previously placed in the table and each odd multiple is calculated by adding two adjacent values in the table, as shown in parentheses in the following illustration.

1	26	
2	52	(2 × 26)
3	78	(26 + 52)
4	104	(2 × 52)
5	130	(52 + 78)
6	156	(2 × 78)
7	182	(78 + 104)
8	208	(2 × 104)
9	234	(104 + 130)
	780	

$$
\begin{array}{r}
6974\\
26\overline{)181345}\\
\underline{156}\\
253\\
\underline{234}\\
194\\
\underline{182}\\
125\\
\underline{104}\\
21
\end{array}
$$

The final item in the table, which is the sum of the last four multiples of the divisor, is used to check the table by comparing it with the third multiple in the table. If the construction of the table is reviewed, it will be seen that the last four multiples depend, as a whole, upon all the previously calculated values in the table and therefore only these last four multiples need be checked. But the sum of 6, 7, 8, and 9 times the divisor is 30 times the divisor. It is extremely unlikely, then, that the table contains an error if the sum of the last four multiples is equal to 10 times the third multiple in the table.

The table of multiples of the divisor is now used to determine the quotient figures and to obtain the required products of the divisor and the quotient figures. The first partial dividend, 181, is selected and compared with the multiples in the table. It is seen at once that 6 is the first quotient figure. The quotient figure is written over the last figure of the partial dividend and the corresponding multiple is written under the partial dividend. After subtraction, the remainder is compared with the divisor to make sure that it is smaller than the divisor. Finally the next digit in the dividend is brought down, and the process is repeated until all the quotient figures have been determined.

When this experimental method of performing long division is used, there are no cases, regardless of the divisor, where there is difficulty in determining the quotient figure; and once the table of multiples of the divisor is constructed, any number of quotient figures can be found with ease. Moreover, this method not only removes trial and error from long division but has other advantages as well. The products of the quotient figures and the divisor are obtained from the table of multiples instead of by multiplication. Hence the only multiplication required in the experimental method is that used in doubling when constructing the table. After the table has been constructed it is possible to concentrate attention on the placing of the quotient figures and partial products, and on the required subtraction.

It may be argued that the experimental method requires more work than the usual method since only a few of the values in the table of multiples may be needed in the division. This is true in solving the simple ready-made long division examples found in arithmetic textbooks. It is not unusual, however, when calculating averages or solving percentage problems in life situations, to find division examples requiring three or four, or even more, quotient figures or having divisors of more than two figures. Unless a division example is simple, the multiplication required in testing trial quotient figures may easily involve as much labor as that required in making a table of multiples of the divisor—or even more. Solving the example at the beginning of this chapter by the usual method requires eleven multiplications, most of which have a multiplier greater than 5. On the other hand the construction of the table requires four easy multiplications by 2 and four easy additions, and the check of the table requires one addition. A systematic table is certainly more useful and is easier to obtain than the collection of separate multiplications often scattered around a division example by pupils who cannot test the trial quotient figures mentally or who cannot multiply easily unless the multiplicand and multiplier are in the normal position.

Solving a group of division examples having the same divisor requires only one table of multiples. The saving of labor and time by using the experimental method in this situation is obvious.

History of the Experimental Method

The method of using a table of multiples of the divisor when performing long division is not new. It can be found in a number of early arithmetics. Recorde[3] (1579) gives the scratch method of division and follows it with the method of using a table of multiples of the divisor. His table is obtained by multiplying the divisor by 2 to 9 in turn. Commenting on this method Recorde says that for large dividends and divisors "there is no better way than this for any man to use, though he be never so experte." He also approves it when the same divisor is used for many different dividends. While Recorde checks the scratch method of division by casting out 9's, he gives no check for the table of multiples.

Hylles[4] (1600) gives the scratch method of division and fol-

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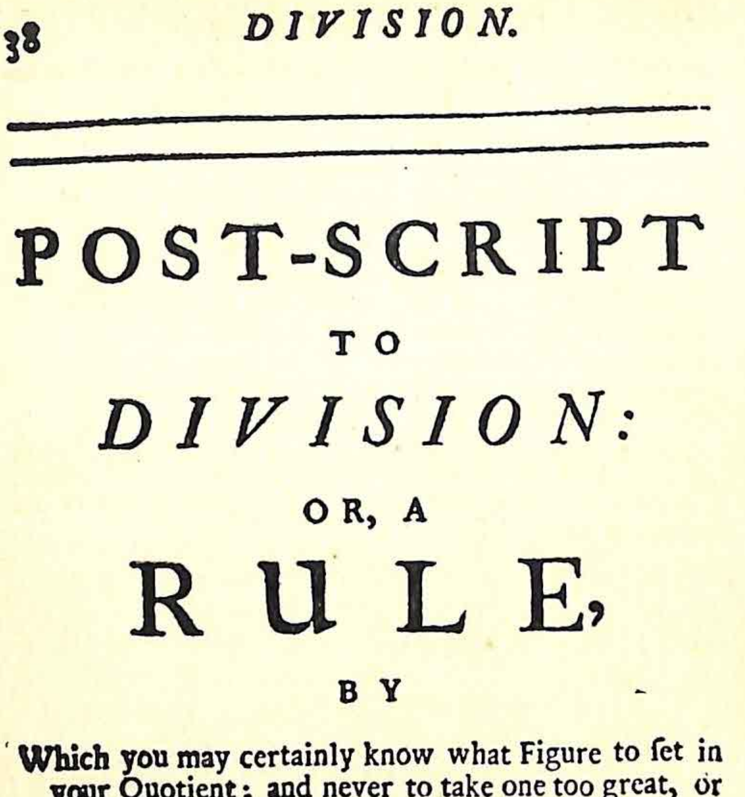
[3] Roberte Recorde, *The Grounde of Artes, teaching the work and practice of Arithmetike, bothe in whole numbers and fractions,* London, 1579 (Original edition, about 1542).

[4] Thomas Hylles, *The Arte of Vulgar Arithmeticke, both in Integers and Fractions,* London, 1600 (Original edition, 1592).



38 *DIVISION.*

POST-SCRIPT
TO
DIVISION:
OR, A
RULE,
BY

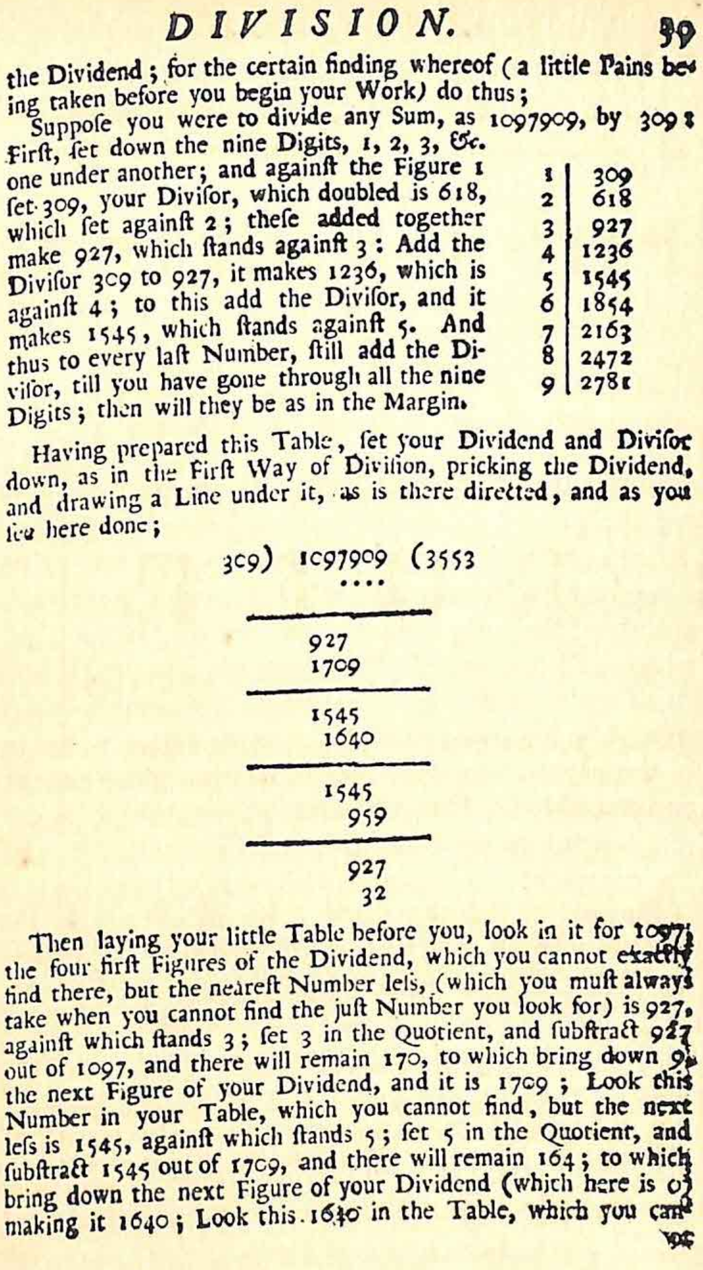
Which you may certainly know what Figure to ſet in your Quotient; and never to take one too great, or too little, but that which will juſt ſerve:

AND

Alſo how to perform (with Eaſe and Certainty) the moſt difficult Sum that can be propoſed in *Diviſion*, without the Aſſiſtance of *Multiplication*; only by *Addition* and *Subſtraction*: Not burthening the Memory at all.

IN the Practice of Diviſion, there is nothing more difficult, than in large Sums (eſpecially if the firſt Figures of the Diviſor be either 1, 2, 3, or Cyphers, and the laſt Figures 7, 8, or 9) to know certainly what Figure to put in the Quotient, when you demand how often the Diviſor may be had in the

Page 38 from William Leybourn's *Arithmetick* (1700)



the Dividend; for the certain finding whereof (a little Pains being taken before you begin your Work) do thus;

Suppoſe you were to divide any Sum, as 1097909, by 309: Firſt, ſet down the nine Digits, 1, 2, 3, &c. one under another; and againſt the Figure 1 ſet 309, your Diviſor, which doubled is 618, which ſet againſt 2; theſe added together make 927, which ſtands againſt 3: Add the Diviſor 309 to 927, it makes 1236, which is againſt 4; to this add the Diviſor, and it makes 1545, which ſtands againſt 5. And thus to every laſt Number, ſtill add the Diviſor, till you have gone through all the nine Digits; then will they be as in the Margin.

1	309
2	618
3	927
4	1236
5	1545
6	1854
7	2163
8	2472
9	2781

Having prepared this Table, ſet your Dividend and Diviſor down, as in the Firſt Way of Diviſion, pricking the Dividend, and drawing a Line under it, as is there directed, and as you ſee here done;

```
309) 1097909 (3553
        ....
     ________
     927
     1709
     ________
     1545
      1640
     ________
      1545
        959
     ________
        927
         32
```

Then laying your little Table before you, look in it for 1097, the four firſt Figures of the Dividend, which you cannot exactly find there, but the neareſt Number leſs, (which you muſt always take when you cannot find the juſt Number you look for) is 927, againſt which ſtands 3; ſet 3 in the Quotient, and ſubſtract 927 out of 1097, and there will remain 170, to which bring down 9, the next Figure of your Dividend, and it is 1709; Look this Number in your Table, which you cannot find, but the next leſs is 1545, againſt which ſtands 5; ſet 5 in the Quotient, and ſubſtract 1545 out of 1709, and there will remain 164; to which bring down the next Figure of your Dividend (which here is 0) making it 1640; Look this 1640 in the Table, which you can

not

Page 39 from William Leybourn's *Arithmetick* (1700) lows it with the method of using a table of multiples of the divisor. Like Recorde, he obtains his table by multiplying the divisor by 2 to 9, and gives no check on the accuracy of the table. After presenting both methods, Hylles summarizes the work on division thus:

> You have now learned two formes of division: The first ordinary and common, the last not so generally used, but more easie, and such as Maister Doctor Recorde thinketh the perfectest and most expert arithmetician need not think scorne at sometimes to use, especially when the dividend and divisor be both great numbers, or when many great numbers are to be divided by one divisor, which thing cometh very often in geometrical and astronomical calculations.

Leybourn,[5] in the seventh edition (1700) of his arithmetic, outlines the present trial-and-error method of obtaining quotient figures and then gives the method of using a table of multiples "by which you may certainly know what figure to set in your quotient; and never to take one too great, or too little, but that which will just serve." Leybourn obtains the table by the continued addition of the divisor to each previously calculated multiple. The first two pages of Leybourn's interesting explanation of the use in division of a table of multiples of the divisor are reproduced here. He completes the solution of his example on the following page of his arithmetic and ends his discussion with the comment: "So is your division ended, and the quotient is $3553\frac{32}{309}$. And with what ease and certainty this is effected, no multiplication being used, I leave the reader to judge."

An arithmetic by John Ward[6] gives an alternate method of multiplication that uses a table of multiples of the multiplicand. This table is constructed by continued addition, and the tenth multiple is included as a check on the accuracy of the table. Later, following the usual method of division, Ward gives the method of using a table of multiples of the divisor in division.

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[5] William Leybourn, *Arithmetick: Vulgar, Decimal, Instrumental, Algebraical,* London, 1700.

[6] John Ward, *Introduction to the Mathematicks,* London, 1713 (Second edition).

In his American arithmetic Greenwood[7] (1729) also explains the use of a table of multiples in both multiplication and division. He obtains the table by continued addition. It is noteworthy that the table of multiples is obtained by multiplication in the earlier books and by continued addition in these later illustrations. Apparently by the time Leybourn wrote it was recognized that the table could be more easily obtained by continued addition than by multiplication.

It is interesting that the use of a table of multiples in division appeared in the Wentworth-Smith *School Arithmetic, Book One*,[8] in 1919, and also in the *Smith-Burdge Arithmetics, Primary Book*,[9] in 1926. In these textbooks the table is obtained by multiplication and is used only when the topic of long division is introduced. In the later book the authors suggest that the pupil use a table only for the first few examples as it will not be needed after that. In later textbooks Smith seems to abandon the use of a table of multiples for introductory purposes in long division.

The method of constructing the table of multiples used in the experimental method of long division under consideration in this study was devised by Clifford B. Upton. Making the table by continued addition certainly seems easier than making it by multiplying the divisor by every number from 2 to 9 inclusive. However, it would probably be difficult to make the table accurately by continued mental addition.

In order to simplify the calculation of the multiples of the divisor Upton developed the procedure described on page 5. He based his plan on the assumption that multiplying by 2 could be easily done mentally, and that the necessary additions could be easily made since in each case two adjacent numbers already in the table are to be added. This plan, then, avoids any writing other than the calculated multiples. One major objection to the usual method of long division is that it requires facility in multiplying by 2 to 9, a facility that some pupils do not have. Another difficulty in the usual method is the estimating and correcting of quotient figures. Upton sought a method of long division that would avoid both of these difficulties. He seriously considered the possibility of obtaining the fifth multiple in the table by annexing a zero to the divisor and then dividing the result by 2, and the possibility of getting the ninth multiple by multiplying the third multiple by 3. After weighing the various possibilities for the calculation of the multiples, the plan of using multiplication by 2 for the even multiples and the addition of two adjacent values in the table for the odd multiples was selected as the best procedure for trial with elementary school pupils.

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[7] Isaac Greenwood, *Arithmetick, Vulgar and Decimal*, Boston, 1729.

[8] G. Wentworth and D. E. Smith, *School Arithmetic, Book One*, p. 184, 1919–1920.

[9] D. E. Smith and H. G. Burdge, *Smith-Burdge Arithmetics, Primary Book*, p. 238, 1926.

The check of the table of multiples was devised and incorporated into the experimental method during the progress of this study. The problem of checking the table seemed at first a difficult matter. If the table were made by continued addition it could be easily checked by adding the first and ninth entries to obtain the tenth multiple of the divisor. This scheme, however, would not check entries 6, 7, and 8 when the multiples were obtained by the procedure used in the experimental method. Therefore the plan of checking the table previously explained was developed.

PROCEDURE IN THIS STUDY

The first part of this investigation consisted of a preliminary study of the experimental method of long division to determine the reactions of teachers and pupils to it in a variety of elementary school situations and to develop satisfactory procedures for teaching it to elementary school children. The experimental method was taught in fourth and fifth grade classes to pupils who had received little or no previous instruction in long division, and to groups of sixth grade pupils who had failed to make satisfactory progress in long division with the usual method. In all groups the reactions of both teachers and pupils were carefully studied. The teaching plan in each group was not fixed in advance but was developed in the light of the reactions of the pupils to the experimental method and the teaching problems encountered. The results of this part of the study are reported in detail in Chapter II.

The preliminary experiences with the experimental method indicated that it was worthy of further study. Therefore an experiment was undertaken for the purpose of comparing in grade five the experimental method and the most promising form of the usual method with respect to both the accuracy of the pupils' performance and the amount of time the pupils used when solving certain kinds of long division examples. Three groups of fifth grade pupils participated in this experiment. The experimental method was taught to two of these groups, and the usual method was taught to the third group. The plan of this experiment and its results are given in Chapter III.

CHAPTER II

PRELIMINARY STUDY OF THE EXPERIMENTAL METHOD OF LONG DIVISION IN TEN GROUPS OF ELEMENTARY SCHOOL PUPILS

BEFORE ATTEMPTING to make a formal comparison of the performance of pupils who have been taught the experimental method of long division with that of pupils who have been taught the usual method, answers to certain questions regarding the experimental method were secured in an informal preliminary study. If pupils and teachers did not like this method of long division, it would not be worth while to study it intensively. Moreover, the initial teaching of a method of long division new to teachers would not be expected to be wholly satisfactory, and therefore the need was felt for teaching experience with the experimental method before planning a comparative study. This chapter reports in detail the preliminary study and presents recommendations relating to the teaching of the experimental method to elementary school pupils.

During the school year 1941–42 the experimental method of long division was taught to ten groups of pupils in the three elementary schools of the Teachers College of Connecticut. Four of these groups were made up of fifth grade children, three of sixth grade children, and three of fourth grade children. The three elementary schools are typical of those commonly found in small industrial cities, and appeared to offer a representative group of elementary school pupils for study.

Immediately after the opening of school in the fall the fifth and sixth grade pupils in these schools were given the test in long division, Test I, shown on page 16. The results of this test were studied, and it was decided to teach the experimental method to four entire classes of fifth grade children. The children in these classes had received little or no previous instruction in long division. After instruction in the experimental method, they were given Test II at mid-year and Test III in June. These tests will be found on page 16. The sixth grade children had all received instruction in long division during the previous year. In each of three sixth grade classes the teacher and the author selected a group of pupils whose scores were sufficiently low to indicate the need for remedial instruction. It was felt that, in general, a pupil needed remedial instruction in long division if he failed to solve correctly four or more of the ten examples in Test I. After a program of instruction in the experimental method of long division these sixth grade children were tested, at mid-year, with Test II. They were also tested again in June with Test III.

In April the experimental method of long division was introduced in three fourth grade classes. The pupils in these classes had, of course, received no previous instruction in long division. In most schools and in most arithmetic textbooks the study of long division does not now begin until the fifth grade. It usually is considered one of the major topics of the fifth grade course of study in arithmetic. However, in order to obtain the reactions of younger pupils to the experimental method, the teachers of these fourth grade classes agreed to begin the topic of long division near the end of the year. Two fourth grade teachers taught the experimental method to their entire classes and one taught it to a selected group of pupils that were more advanced in their study of arithmetic than the remainder of the class. Although it was too early to attempt to measure the final achievement in long division of these pupils, Test III was given to these groups in June in order to study their progress.

Even though the preliminary study was in no sense a controlled experiment, it seemed desirable to obtain some basis for making a rough evaluation of the achievement of the pupils taught the experimental method. Hence Test III was also given, as was reported in Chapter I, at the end of the year to 520 fifth grade pupils in a number of elementary schools. In order to limit the school time needed to test these pupils, Test III was constructed with only five division examples.

Test I

$21\overline{)76818}$ $12\overline{)42948}$ $79\overline{)99935}$ $53\overline{)498518}$
$17\overline{)44157}$ $32\overline{)47552}$ $28\overline{)57918}$
$36\overline{)85320}$ $69\overline{)91941}$ $274\overline{)105764}$

Test II

$31\overline{)86614}$ $12\overline{)75528}$ $59\overline{)82423}$ $63\overline{)454167}$
$18\overline{)46697}$ $23\overline{)36478}$ $38\overline{)79239}$
$26\overline{)74360}$ $77\overline{)99728}$ $283\overline{)140368}$

Test III

$21\overline{)8337}$ $68\overline{)42787}$ $54\overline{)27458}$
$87\overline{)417252}$ $174\overline{)49653}$

At the beginning of the year the author described the experimental method of long division to the teachers and discussed with them various approaches to the teaching of it. Since neither the teachers nor the author had had previous experience in teaching this method to children, the teaching procedure was not fixed in advance but was left a flexible matter in each group. All the teachers were encouraged to experiment freely in ways of teaching the experimental method to the pupils in their classes. This freedom from a prescribed procedure aided in achieving one of the principal aims of this preliminary study, that is, finding satisfactory ways of teaching the experimental method.

Throughout the year the reactions of the pupils were carefully watched by the teachers and the author, and modifications of teaching procedures were made when it seemed to be desirable in any group. In some groups the pupils were given mimeographed sheets containing thirty-two tables of multiples of a variety of 2-figure numbers.[1] They were then taught to use these tables in solving long division examples having these numbers as divisors. After they had mastered the use of the tables, they were taught to make their own tables, using the short method explained in Chapter I, and the mimeographed sheets were discarded. This procedure was used in order to teach one new thing at a time and, in remedial work, to give the pupil the stimulation of immediate success in a topic with which he had become discouraged. For use in the beginning instruction, the teachers in all groups were provided with a set of long division examples having 3-figure and 4-figure quotients without zeros and having no remainders. To assist the teacher in locating pupil difficulties a short diagnostic test in addition, subtraction, and multiplication was given before instruction in long division began.

The fifth grade teachers were asked to devote to long division no more time than they normally devoted to this topic when teaching the usual method. These teachers also included in their arithmetic programs all of the other regular topics of fifth grade arithmetic. Because of the exploratory nature of this preliminary study it seemed undesirable to attempt to establish a uniform or rigid period for instruction in long division. The teachers reported, however, the average amount of time used per week for long division, together with the number of weeks of instruction. In most of the groups this reported time included all school work in long division and represented the time spent in the following activities: presentation of subject matter by the teacher, group discussion, solution of division examples by pupils at their seats and at the blackboard, analysis of pupil errors, individual instruction, testing, and necessary remedial work in addition, subtraction, and multiplication. In a few of the groups the pupils were given assignments of additional division examples to be solved in free time during the school day.

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[1] After the preliminary study the number of prepared tables on this sheet was reduced to 20. The revised sheet will be found on page 40.

No time limit was set in the administration of Test I in the fall and Test II at mid-year. The pupils were encouraged to work as efficiently as possible and were directed to hand their papers to the teacher as soon as they had completed them. The teacher made a record of the time used by each pupil for the test. Test III, given at the end of the year, was administered in the same way, except that it was suggested that the teachers stop the pupils at the end of 35 minutes.

The author did not feel justified in requiring the checking of division, but left that matter in the hands of each teacher. In those classes where checking was a regular part of all arithmetic instruction the pupils checked their division examples by multiplication both in their daily work and on the tests. Some of the differences in the time used by the pupils for the tests are undoubtedly due to a variation in the amount of checking. At this stage of the study the method of checking the accuracy of the table of multiples, described in Chapter I, had not yet been devised and therefore the pupils did not check their tables during this preliminary study. The need of a check for the table of multiples was recognized at this time.

At the end of the year the teachers were asked to make a report of their reactions to the experimental method, as well as those of their pupils, and to make suggestions for teaching it successfully.

Since the preliminary study of the experimental method took place in a variety of situations in grades four, five, and six, the experiences and results in each group will now be reported in detail.

FIFTH GRADE GROUPS

In the following four groups of fifth grade pupils the experimental method of long division was tried out at the usual time for instruction in long division in the elementary school. The pupils in these groups had received very little or no previous instruction in long division. In each case the experimental method was taught to all the pupils in the teacher's fifth grade class.

Group 5A

At the beginning of the year there were 35 pupils in Group 5A but by mid-year the number had increased to 40. The intelligence quotients ranged from 84 to 132 with a mean of 104.0. When these pupils were given Test I in September, 30 of them made a score of zero. Since it was obvious from their papers that almost all of these pupils had no understanding of long division, it was decided to teach the experimental method to the entire class.

The teacher of this group decided not to use the mimeographed sheets of prepared tables. Instruction began with an explanation of long division in which a table of multiples of the divisor prepared by the teacher was used. The pupils were immediately taught how to make such a table by the short method, and thereafter in doing long division examples they made their own tables of multiples of the divisors. Before the mid-year test the group spent an average of 90 minutes a week on long division for 16 weeks. After the test, instruction was continued for 5 weeks with an average of 45 minutes per week devoted to it.

The results of Test I and Test II are shown in Table 3. By mid-year, when Test II was given, this group was doing excellent work in long division. Half of the group had perfect papers and 85 per cent of the class solved correctly at least seven of the ten test examples. The mean time used for Test II was 65 minutes. This included time for checking each example. The teacher of the group was interested in finding out what the results would be if the pupils were not required to check their work, so she repeated Test II a month later, directing the children not to check. The mean time dropped to 37 minutes but accuracy was reduced only slightly. Checking by multiplication is unquestionably a time-consuming process.

In June 35 of the pupils of this group took Test III, and made an excellent record of achievement in long division. The scores are shown in Table 4. In this group 43 per cent of the pupils solved all five test examples correctly and 83 per cent solved at

TABLE 3

Distributions of Scores of Fifth Grade and Sixth Grade Groups on Test I and Test II

No. of Examples Correct	*Group 5A*		*Group 5B*		*Group 5C*		*Group 5D*		*Group 6A*		*Group 6B*		*Group 6C*	
	I	*II*	*I*	*II*	*I*	*II*	*I*	*II*	*I*	*II*	*I*	*II*	*I*	*II*
10		20	1	11		7		5		3		7		
9		9		4		7	2	4		3		1		2
8		3	1	5		3		2		5		3		
7	2	2	3	3		5		1		2		3		1
6		2	2	6		2			5	1	2			3
5		1				4		2		5	2			2
4			1			1	3	3	6	2	2		3	1
3	1		4	1		2	2		3		5	1	4	3
2	1	1	3	1		1	1		1	1			3	1
1	1		6			1	1		3		2		4	2
0	30	2	10		29		3		5	1	2		1	
Total	35	40	31	31	29	33	12	17	23	23	15	15	15	15

least four examples correctly. The mean time used by Group 5A for Test III, including checking, was 31 minutes. Of these pupils, 69 per cent turned in their papers during the final 5-minute interval of the 35 minutes allowed.

The teacher of Group 5A was enthusiastic about the experimental method. She reported that the pupils liked this method of long division and that it seemed easy for them to understand. She also said that the pupils "mastered long division in a much shorter time than under the old method" and that she was "able to discover more quickly just where children need remedial work." The progress of this group was unusually rapid and both the teacher and the author felt that the pupils' achievement in long division was superior.

Group 5B

Group 5B consisted of 31 fifth grade pupils whose intelligence quotients ranged from 72 to 129 with a mean of 108.4. These children had received some previous instruction in long division in the fourth grade. Although long division is regularly a fifth grade topic, it is occasionally introduced at the end of the fourth grade to pupils of high achievement in arithmetic. After the teacher had solved a few illustrative long division examples on the blackboard for review, she gave Test I. Since the results of this test indicated that most of the pupils had made little progress in long division, it was decided to teach the experimental method to the entire class.

The teacher began instruction by placing a table of multiples on the blackboard and showing its use in performing long division. After the children had solved some examples by using this table on the blackboard they were given the mimeographed sheets of prepared tables. They soon mastered the use of these tables and were then taught to make tables by the short method. Thereafter they made their own tables for long division examples. Before the mid-year test this group devoted approximately 150 minutes per week to long division for 12 weeks. During the second part of the year they spent an average of 75 minutes per week on long division for 12 weeks.

S.C.E.R.T., West Bengal
Date..........................
Acc. No. 819..............

TABLE 4

DISTRIBUTIONS OF TEST III SCORES OF ALL GROUPS IN GRADES FOUR, FIVE, AND SIX

No. of Examples Correct	*Group 5A*	*Group 5B*	*Group 5C*	*Group 5D*	*Group 6A*	*Group 6B*	*Group 6C*	*Group 4A*	*Group 4B*	*Group 4C*
5	15	10	15	4	3	7	1	1	2	
4	14	11	8	5	9	4		2	1	1
3	3	3	5	2	2	2	4	5	4	1
2		5		3	5		5	5	2	3
1	1		3	1	1	1	3	7	3	2
0	2		1	2	1		2	2		9
Total	35	29	32	17	21	14	15	22	12	16

The results of Test I and Test II for Group 5B are shown in Table 3. At mid-year, when Test II was given, 74 per cent of the group solved at least seven of the ten test examples correctly. The mean time for Test II was 38 minutes. Two pupils completed perfect papers in 22 minutes. In June Test III was given to 29 of the pupils of the group, with the score results shown in Table 4. Of these pupils 34 per cent solved all five test examples correctly and 72 per cent solved at least four examples correctly. The mean time used for Test III by Group 5B was 31 minutes. When taking this test almost all of the children checked their work. During the last 5-minute period of the 35 minutes allowed for the test, 62 per cent of the group handed in their papers.

In her report the teacher of this group said that the experimental method "gives a clearer meaning to division" and "eliminates fear" of division. She believed that the pupils liked this method of long division but that it became, toward the end of the year, "monotonous for the brighter pupils." She suggested that instruction in long division begin with the experimental method for all pupils and that the more capable pupils, after having learned it, be taught the usual method. She would refer these more capable pupils to the experimental method when they were in difficulty. For the slow pupil this teacher believed the experimental method to be excellent.

Group 5C

Group 5C was an entire fifth grade class of 33 pupils. Their intelligence quotients ranged from 69 to 130 with a mean of 97.8. All of these pupils made a score of zero on Test I and only one pupil even made an intelligent attempt to solve any of the test examples. They apparently had received little, if any, previous instruction in long division.

The teacher began instruction by illustrating the use of a written table of the multiples of 9 in solving a division example having 9 as the divisor. She then proceeded to division with a 2-figure divisor, putting a table of the mutiples of 14 on the blackboard. After the pupils had used this table in solving a number of division examples, the mimeographed sheets of prepared tables were distributed to them. As soon as the children showed proficiency in using these sheets in long division they were taught to make a table of multiples. There was considerable discussion at the beginning of this topic of various ways of constructing a table. The class first made a table by multiplication and by continued addition. In developing the short method the children were led to see that, in finding 9 times a given number, adding 4 and 5 times the number is more convenient than adding 3 and 6 times the number. All of this discussion gave the children a thorough understanding of the meaning of a table of multiples. When they were making tables they often placed a finger at the side of the two numbers to be added in order to avoid the error of adding the wrong numbers. The teacher found that timing the children while they were making tables reduced errors due to lost places. As the pupils grew in ability to make tables they began to do long division examples, making their own tables. Before the mid-year test the group received an average of 60 minutes of instruction in long division per week for 16 weeks. During the remainder of the year there was one 25-minute period of instruction per week for 12 weeks. These pupils received some additional assignments to be prepared during the school day.

The results of Tests I and Test II for Group 5C are given in Table 3. At mid-year 67 per cent of the group solved correctly at least seven of the ten examples on Test II. The mean time for Test II, with checking, was 65 minutes. In June Test III was given to 32 of the children in this group, with the score results shown in Table 4. In this group 47 per cent solved all examples correctly and 72 per cent solved correctly at least four of the five test examples. The test papers in the group were collected by the teacher at the end of 40 minutes. The mean time was 30 minutes and half of the group used more than this amount of time. All of the pupils checked their work by multiplication.

The teacher reported that the pupils reacted very favorably to the experimental method and that "doing away with the trial and error made for fewer disgruntled children and greater speed in accomplishment." She said that she thought the experimental method was easier to teach and that she liked very much the plan of separating the teaching of the long division process from the making of tables. This teacher believed that one of the major problems in teaching long division is checking, and that multiplication is not a satisfactory procedure since the pupil, when his example fails to check, is uncertain whether to look for an error in the division or in the work of the check.

Group 5D

Group 5D was a small fifth grade class of 17 pupils. Their intelligence quotients ranged from 93 to 124 with a mean of 107.1. Twelve of these pupils had received previous instruction in long division, and were given Test I after having a brief review of long division. Only two of them solved correctly more than four of the examples on this test, and it was decided to teach the experimental method to the entire class.

The teacher began instruction by explaining the long division process, using a table of multiples on the blackboard and the mimeographed sheets of tables. The children were given a few days' practice in long division with these prepared tables and then they were taught to make the tables the short way. Thereafter they made their own tables for long division examples. The group received approximately 12 weeks of instruction consisting of two 30-minute periods a week before the mid-year test. After this test they received the same amount of instruction per week for 12 more weeks.

The results of Test I and Test II for Group 5D are given in Table 3. At mid-year 71 per cent of the class solved correctly at least seven of the ten examples on Test II. The mean time used by Group 5D for Test II was 80 minutes. This included time for checking each example. This group did not achieve the speed with the experimental method of some of the other groups. In June Test III was given to this group, with the score results shown in Table 4. Of the pupils in the group 24 per cent solved all five examples correctly and 53 per cent solved at least four examples correctly. All but one pupil used all of the available time for Test III, 35 minutes, and all checked their work.

The teacher of this group reported that some of the pupils liked the experimental method but that others felt it took too much time. She stated that "the strongest reason for my liking the new method is that even the slowest children can learn to do long division. For the bright children it seems a bit cumbersome."

REMEDIAL GROUPS IN GRADE SIX

The experimental method was used for remedial instruction with three groups of sixth grade pupils. All of the pupils in these groups had previously been taught the usual method of long division but they failed to show satisfactory proficiency in this work when tested at the beginning of the year. Each remedial group was a part of the teacher's regular sixth grade class.

Group 6A

Group 6A consisted of 23 sixth grade children. At the beginning of the year no child had solved correctly more than six of the ten examples in Test I. The intelligence quotients of the group ranged from 83 to 114 with a mean of 98.8.

The teacher began the remedial instruction by explaining the meaning of a table of multiples and giving the mimeographed sheets of tables to the pupils. She then taught the use of these tables in performing long division. After the children had been given considerable practice in using tables they were taught to make their own tables by the short method. Some learned how to make a table at once and a few had difficulty for a long time.

Before the mid-year test the entire group received 12 weeks of instruction consisting of 70 minutes a week. In addition the slower pupils received an extra 30-minute period a week. The results of Test I and Test II for Group 6A are given in Table 3. In this group 57 per cent of the pupils were able at mid-year to solve at least seven of the ten test examples correctly. The individual gains of these pupils are shown in Table 5. All but two pupils improved their scores. The teacher was particularly pleased with the gains of some of the slow pupils who had previously made no progress in long division. The mean time of the

TABLE 5

DISTRIBUTIONS OF GAINS FROM TEST I TO TEST II IN SIXTH GRADE GROUPS

Number of Examples Gained	*Group 6A*	*Group 6B*	*Group 6C*
10		1	
9	1	2	
8			1
7	1	1	1
6	3	1	
5	2	4	2
4	8	2	
3	2	3	
2	2	1	6
1	2		2
0			1
−1	1		1
−2			1
−3	1		

group for Test I was 37 minutes. However, at the time Test I was given several pupils were so poor in long-division that they either did not attempt the harder examples or left some examples incomplete. The mean time for Test II was 58 minutes, including checking. One pupil solved all 10 examples correctly in 25 minutes.

During the second part of the year this group received very little additional instruction in long division. The pupils who were doing good work received a written assignment every two weeks and those having difficulty had a period of instruction every two weeks. In June Test III was given to 21 members of the original group, with the results found in Table 4. In Group 6A, 14 per cent solved all five examples correctly and 57 per cent solved at least four examples correctly. Almost all of the pupils checked their work. For Test III this group was limited by the teacher to 45 minutes and the mean time used was 30 minutes. The pupils in this classroom who were not receiving remedial instruction also took Test III "for fun." It was interesting to find that a few of these pupils used the experimental method on this test. Three pupils made a table for all examples and one made a table only for the example having a 3-figure divisor.

The teacher reported that the majority of the children were "very, very enthusiastic about the method." She said that she liked the experimental method because it gave confidence to the pupils, made them more independent of the teacher, and "took away a goodly amount of fear of long division." Giving these pupils the prepared tables of multiples seemed a wise plan, for the immediate success in a topic that had been a stumbling block created an interest in arithmetic.

Group 6B

Group 6B was made up of 15 children. At the beginning of the year no child had solved correctly more than six of the ten examples on Test I. The intelligence quotients ranged from 89 to 122 with a mean of 101.6.

The remedial instruction began with an explanation of the use of a table of multiples of the divisor in performing long division. This was followed by practice in using the mimeographed sheets of tables. The pupils were then taught to make their own tables by the short method. While learning to make a table these pupils constructed a large chart showing the procedure as follows:

1 (Divisor)
2 (double 1)
3 (add 1 and 2)
4 (double 2)
5 (add 2 and 3)
6 (double 3)
7 (add 3 and 4)
8 (double 4)
9 (add 4 and 5)

The teacher felt that this chart was an effective aid to the pupils in learning the short method of making the table.

The pupils in Group 6B received 3 weeks of group instruction of approximately 60 minutes a week. Thereafter individual help was given as needed. The results of Test I and Test II are shown in Table 3. At mid-year 93 per cent of the group solved correctly at least seven of the ten test examples. As indicated in Table 5, all of the pupils in this group improved their scores. The mean time for Test I was 44 minutes. As in Group 6A, several pupils, when taking Test I, either did not attempt some of the examples or left examples incomplete. The mean time for Test II was 63 minutes. Both the teacher and the author felt that the remedial program was highly successful.

During the second part of the year there was no additional instruction in long division. In June Test III was given to 14 members of this group, with the score results in Table 4. Of these pupils 50 per cent solved all five examples correctly and 79 per cent solved at least four examples correctly. The time used by the group was limited to 35 minutes and the mean time was 27 minutes.

The teacher reported that this was the first year that the slow pupils in her class had been able to make progress in long division. She said that "previously, in Self-Testing Drills many children simply skipped the long division examples and didn't even attempt them. Since this new method was presented to the children, not one child skipped a long division example." She also said that she found the experimental method a great convenience and time-saver for herself in the preparation of the statistical material in school reports.

Group 6C

Group 6C consisted of 15 pupils selected for remedial instruction in long division from a sixth grade class of 35 children. The pupils selected for this group were those who had solved correctly not more than four of the ten examples on Test I. Their intelligence quotients ranged from 71 to 120 with a mean of 95.6.

The pupils were first given the mimeographed sheets of prepared tables and were taught to use them in performing long division. After practice in the use of these tables the children were taught to make a table by the short method. Thereafter in doing long division they made their own tables.

The remedial instruction consisted of four 30-minute periods a week for 10 weeks. The results of Test I and Test II are shown in Table 3. At the mid-year only 20 per cent of the group achieved a score of seven correct examples. The individual gains are shown in Table 5. This group's performance was somewhat better on Test II than on Test I but only 4 pupils substantially improved their scores. The mean time for Test I was 33 minutes and for Test II was 40 minutes.

During the second part of the year the instruction in long division was only incidental when division problems arose. In June Group 6C made the scores on Test III shown in Table 4. The time was limited to 35 minutes and the mean time used was 20 minutes. Only one pupil, or 7 per cent of the group, solved more than three examples correctly. The teacher also gave Test III to the 20 pupils in the class who had not participated in the remedial program and only 9, or 45 per cent, solved more than three examples correctly.

In her report concerning the experimental method this teacher stated: "As I see it now I do not feel that the end accomplished warranted the time placed on it." She believed that the pupils made as many computational errors when using the experimental method as they had been making with the usual method.

FOURTH GRADE GROUPS

In the following three groups of fourth grade pupils the study of the experimental method of long division began in April and continued for the remainder of the school year. Two teachers taught it to their entire fourth grade classes, and one teacher taught it to a part of her fourth grade class.

Group 4A

Group 4A was an entire class of 22 fourth grade children. Their intelligence quotients ranged from 77 to 122 with a mean of 103.7. They had received no previous training in division with 2-figure divisors.

The study of the experimental method began in April, and before the end of the school year the group received about two months of instruction. The pupils devoted about 4 hours per week of their school time to long division. The teacher first taught them to make tables of multiples of numbers. After the class understood this process thoroughly, she then taught division with a 2-figure divisor, using a table of multiples of the divisor. This group did not use the mimeographed sheets of prepared tables but made their own tables for division examples from the beginning. To reduce errors in addition when making a table the teacher found it helpful to have the pupils place two fingers of the left hand at the side of the two numbers to be added, and to have them add a carry number immediately to one of the two digits in the next column rather than add it to the sum of these digits.

In June the group was given Test III, with a time limit of 35 minutes. Their scores are shown in Table 4, and the mean time used was 18 minutes. These pupils had not, of course, mastered long division in this short period of instruction. Only 14 per cent of the group solved correctly at least four of the five test examples. However, their papers showed that all but one pupil had a thorough understanding of the long division process. Lack of success was due to errors in addition and subtraction.

The teacher of this group liked the experimental method very much and expressed the opinion that it eased the task of teaching long division immeasurably. In her report she said: "I've struggled with the teaching of long division for many years but never has it been mastered as quickly or as efficiently by any average group as it has by this group. In former years children, especially slow groups, needed endless individual help and then didn't really understand." She also said that the pupils liked long division and asked for more work than she gave them. Very soon after instruction began this group was doing division examples of almost any difficulty. The experiences with this group seem to indicate that the experimental method offers a superior way to introduce the topic of long division. The teacher also believed that the experimental method helped to develop number concepts.

Group 4B

Group 4B consisted of 13 pupils selected from a fourth grade class. They were more advanced in the study of arithmetic than the remainder of the class, but had received no instruction in long division. One of these pupils had apparently learned the usual method outside of school. Their intelligence quotients ranged from 110 to 137 with a mean of 123.9.

The teacher introduced the experimental method of long division in the middle of April and the group received 8 weeks of instruction of approximately 60 minutes per week. The teacher did not give the pupils the mimeographed sheets of prepared tables. She taught the long division process and the making of tables together. Under her guidance the pupils worked out the short method of constructing the table for themselves. It was interesting to note that the need for the short method of making the table was more apparent to these pupils for a divisor such as 38 than for one like 21.

The scores of Test III, which was given in June, are shown in Table 4. Thirty-five minutes were allowed for this test, and the mean time used was 24 minutes. One pupil did not make tables for Test III and is not included in these results. All of the children, with the exception of this one pupil, were eager to learn the experimental method. This pupil felt that she already knew long division and showed no interest in this different method. However, she solved only one example correctly on Test III. The group did not achieve great accuracy in long division. Only 25 per cent of the pupils solved at least four of the five test examples correctly. The test papers showed that every pupil understood this method of long division; almost all of the errors were in addition and subtraction.

The teacher said that she liked the experimental method and that she thought it "easier to explain." From the point of view of the pupils she believed it "more readily comprehended" and "more enjoyable." She thought it possible to progress more quickly to the solution of difficult long division examples with this method than with the usual method.

Group 4C

Group 4C was a small fourth grade class of 16 pupils. Their intelligence quotients ranged from 93 to 129 with a mean of

108.5. They had received no previous instruction in long division.

The experimental method of long division was introduced to this group in April and the pupils received two months of instruction of approximately 80 minutes a week. The teacher began instruction in performing long division by making tables of multiples on the blackboard and showing their use in the solution of long division examples. The children were then given practice in using the tables that had been made on the blackboard and that were on the mimeographed sheet. After the teacher felt that the pupils understood the use of tables of multiples she taught the group to make them by the short method. Thereafter the children made their own tables for most of their work in division. For some of their practice work, however, they continued to use the sheets of prepared tables.

In June this group was given Test III, with a time limit of 35 minutes. The scores are given in Table 4, and the mean time was 32 minutes. The pupils made a very poor showing on this test. An examination of the test papers, however, showed that one-half of the group had a thorough understanding of the process of long division and had made errors only in addition and subtraction. The teacher of the group reported that the children liked to do long division by this method and that they used some of their free time to make tables and solve division examples of their own invention. She thought that the group made good progress in long division. The author believes that one of the reasons for the low scores on Test III was the limited experience of the pupils in the independent solution of division examples.

The teacher said that the experimental method was of much benefit to herself and that now her "annual report and register are accurately and painlessly done."

SUMMARY OF PRELIMINARY STUDY

Fifth Grade Groups

The children in the fifth grade groups reacted very favorably to the experimental method and their achievement in long division seemed highly satisfactory. At mid-year 75 per cent of the 121 pupils in these four groups solved correctly at least seven of the ten examples on Test II. When 113 of these pupils took Test III in June, 39 per cent of them solved correctly all five examples, and 73 per cent solved correctly at least four of these examples. For the 520 fifth grade children taught the usual method of long division, under average conditions, the corresponding figures for Test III were 18 per cent and 39 per cent. A summary of the fifth grade test results is given in Table 6.

There was considerable variation in the amount of time used for the long division tests by the pupils in the fifth grade groups. Some pupils made tables very rapidly and others were slow. There was also lack of uniformity in the amount of checking, although almost all of the pupils in these groups checked their work. When Test III was given, 65 per cent of the pupils used more than 30 minutes for this test. The corresponding figure for the 520 fifth grade pupils in other schools was 52 per cent. However, only about one-third of these pupils checked their work. From the study of test results, observation of pupils, and reactions of teachers, it seemed that the time required for using the experimental method is not seriously greater than that required for using the usual method.

All four teachers of the fifth grade groups liked the experimental method. In their opinion this method of long division is easier for pupils to understand than the usual method. They all approved the experimental method for the slow pupils and thought their achievement in long division when using it justified any additional time required for making a table. Two of the teachers thought it somewhat long for the more able pupils when solving easy examples, and one of these teachers suggested that the usual method be taught to these brighter pupils in addition to the experimental method.

The advantages of the experimental method seem to increase with the difficulty of the division example. The more quotient figures that are required in an example, the more useful the table of multiples of the divisor. If the divisor has more than two figures the estimation of quotient figures, when the usual

TABLE 6

SUMMARY OF FIFTH GRADE AND SIXTH GRADE TEST RESULTS (PRELIMINARY STUDY)

Group	*Per Cent of Pupils Solving Correctly at Least 7 Test II Examples*	*Per Cent of Pupils Solving Correctly All 5 Test III Examples*	*Per Cent of Pupils Solving Correctly at Least 4 Test III Examples*
5A	85%	43%	83%
5B	74	34	72
5C	67	47	72
5D	71	24	53
All four fifth grade groups	75	39	73
520 pupils in other schools		18	39
6A	57	14	57
6B	93	50	79
6C	20	7	7

method is employed, becomes more difficult and the multiplication required gives increased opportunity for error. There was one example on Test III having a 3-figure divisor. It was found that 57 per cent of the 113 pupils in the fifth grade groups solved this example correctly while only 29 per cent of the 520 fifth grade pupils in other schools solved it correctly. If a situation requires the solution of a group of division examples having the same divisor, the experimental method saves both work and time. It was interesting that several teachers, some of whom were not participating in this study, reported to the author that they found the experimental method a great time-saver in the preparation of school reports and statistics when the calculation of percentages by long division was required.

Sixth Grade Groups

The pupils in the three sixth grade groups had all been previously taught the usual method of long division, and the experimental method was used only for remedial instruction.

In two of these groups the remedial program was very successful. The results in these two groups show the possibilities of the experimental method in remedial instruction. In the fall no pupil in the groups was able to solve correctly more than six of the ten examples in Test I. At mid-year 56 per cent of Group 6A and 93 per cent of Group 6B reached a score of seven correct examples out of the ten examples in Test II. All but two pupils in these two groups improved their scores from Test I to Test II and most of them made a substantial gain. In June the results for Test III showed excellent retention of the skill developed by the remedial programs. In Group 6A, 57 per cent of the pupils solved correctly at least four of the five test examples, and in Group 6B, 79 per cent of the pupils reached this achievement. In these sixth grade groups, as in the fifth grade, the time used for the long division tests varied widely. Some pupils used the method with remarkable speed; some were very slow.

The children in these two groups appeared to like the experimental method and to enjoy their success in a topic that had been discouraging and difficult. The two teachers were enthusiastic about the experimental method and approved its use in remedial instruction. In their opinion it removed fear of long division and made possible a higher achievement in this topic for the less capable pupils. They believed that the success of the slow pupils with it justified the time required for them to make the table of multiples of the divisor.

In the other sixth grade group the remedial program was not very successful. Most of the pupils in this group improved their scores very little from Test I to Test II, and the group made a poor showing on Test III. The pupils had no difficulty in understanding the experimental method but made many errors in addition and subtraction. The teacher of this group considered the frequency of errors in addition to be one of the major disadvantages of this method. The results in this group emphasize the importance and necessity when teaching long division of the elimination of errors in prerequisite arithmetical processes.

A summary of the sixth grade test results is given in Table 6.

Fourth Grade Groups

Long division is not a normal topic for grade four but the experimental method was introduced in these classes to test its usefulness to these younger pupils and to obtain additional information in regard to its value in the introductory instruction in long division.

The fourth grade groups received about two months of instruction in long division. The results on Test III show for these groups limited achievement in the performance of long division with accuracy. However, an examination of the test papers indicated that practically all the pupils in two of these groups and half the pupils in the third group had achieved a complete understanding of this process of long division. It was clear that they knew how to make a table of multiples by the short method, to select the partial dividend and compare it with the multiples in the table in order to obtain the correct quotient figure, to place the quotient figure properly, to write the corresponding multiple under the partial dividend and subtract, and to bring down the next digit in the dividend to make the next partial dividend. But these pupils made errors in addition when constructing the tables and errors in subtraction when carrying out the division. They also understood how to handle remainders and zeros in the quotient. All that remained in the teaching of long division to these pupils was the elimination of numerical errors. That the experimental method is easy for children to understand was indicated by the fourth grade experiences.

The fourth grade pupils who understood the experimental method used it rapidly. In Group 4A all but one pupil completed Test III in 25 minutes or less. The pupils in the fourth grade groups did not, however, check their work by multiplication when taking this test.

With one exception, the pupils were eager to learn the experimental method of division. They liked to make the tables, and in one group the teacher had no difficulty in interesting the pupils in learning to make tables of multiples of numbers before she showed their use in long division. All three teachers of these fourth grade groups said they liked the experimental method. They believed that it was very easy for the pupil to understand, that it made possible the solution of division examples of any difficulty after a short period of instruction, and that the pupils enjoyed using it. Since the procedures in the experimental method seem to be much more clearly understood by pupils than those of the usual method, the experimental method appears to be of value in the beginning instruction in long division.

RECOMMENDATIONS RELATING TO TEACHING THE EXPERIMENTAL METHOD

One of the major purposes of the preliminary study of the experimental method was to secure suggestions on how to teach it most effectively to elementary school children. The recommendations given later in this chapter are the outcome of the experiences, in all the groups, of the teachers and the author in working out the difficulties and teaching problems that were encountered throughout the preliminary study.

Each group of pupils in the preliminary study was taught the experimental method by using one of the following three plans:

Plan I. In this plan the pupils were first shown how to solve an example in long division by using a table of multiples of the divisor. The mimeographed sheets of prepared tables of multiples were then distributed and the pupils were taught to use these tables in long division. As soon as the pupils were proficient in the use of the mimeographed tables, which happened in a surprisingly short time, they were taught to make a table of multiples by the short method. The mimeographed sheets of prepared tables were then discarded, and in succeeding work in long division the pupils made their own tables.

Plan II. In this plan instruction in long division began with a demonstration of long division in which the teacher made use of a table of multiples of the divisor that she had placed on the blackboard. With the purpose of a table of multiples shown, its construction by the short method was immediately presented. Thereafter the pupils made their own tables of multiples of the divisors when solving division examples, and the mimeographed tables were never used.

Plan III. In this plan the pupils were first taught to make tables of multiples of numbers by the short method. This instruction preceded any mention of long division or of the use of a table of multiples in long division. After the pupils became proficient in the construction of tables, the experimental method of long division was introduced. These pupils were ready to make their own tables of multiples of the divisor from the beginning, so the mimeographed sheets of tables were, of course, not used.

For remedial instruction in long division Plan I seems to be a wise procedure. The pupils in the sixth grade groups learned quickly to use the mimeographed tables in the solution of examples in long division. This success, which created a new interest in arithmetic and developed self-confidence in these pupils, was effectively used to motivate the study of the construction of a table of multiples.

For the instruction of pupils studying long division for the first time all three of these plans were used satisfactorily. Plan III, however, was used in only one group, which was a fourth grade class. In this group the pupils were very eager to learn to make tables of multiples of numbers even though they did not know the purpose of making them. The success of this plan will undoubtedly depend upon whether or not the pupils take an interest in the construction of tables of multiples of numbers as such. Most of the teachers with whom the author has discussed these plans prefer Plan I or Plan II.

When mimeographed sheets of prepared tables are to be used it is desirable that the number of tables placed on the sheet be sufficiently small to provide a blank space between adjacent tables. The tables were so close together on the sheet used in the preliminary study that a multiple from the wrong table was frequently copied. The period of time during which the prepared tables are used will be short; hence a limited number of tables will be adequate. It is recommended that the teacher use a sheet of prepared tables similar to the one shown on page 40. This sheet of tables was used in the final part of this study. When cut-

Sheet of Prepared Tables of Multiples

14	
1	14
2	28
3	42
4	56
5	70
6	84
7	98
8	112
9	126

17	
1	17
2	34
3	51
4	68
5	85
6	102
7	119
8	136
9	153

23	
1	23
2	46
3	69
4	92
5	115
6	138
7	161
8	184
9	207

26	
1	26
2	52
3	78
4	104
5	130
6	156
7	182
8	208
9	234

28	
1	28
2	56
3	84
4	112
5	140
6	168
7	196
8	224
9	252

34	
1	34
2	68
3	102
4	136
5	170
6	204
7	238
8	272
9	306

37	
1	37
2	74
3	111
4	148
5	185
6	222
7	259
8	296
9	333

41	
1	41
2	82
3	123
4	164
5	205
6	246
7	287
8	328
9	369

44	
1	44
2	88
3	132
4	176
5	220
6	264
7	308
8	352
9	396

46	
1	46
2	92
3	138
4	184
5	230
6	276
7	322
8	368
9	414

52	
1	52
2	104
3	156
4	208
5	260
6	312
7	364
8	416
9	468

58	
1	58
2	116
3	174
4	232
5	290
6	348
7	406
8	464
9	522

62	
1	62
2	124
3	186
4	248
5	310
6	372
7	434
8	496
9	558

65	
1	65
2	130
3	195
4	260
5	325
6	390
7	455
8	520
9	585

73	
1	73
2	146
3	219
4	292
5	365
6	438
7	511
8	584
9	657

79	
1	79
2	158
3	237
4	316
5	395
6	474
7	553
8	632
9	711

84	
1	84
2	168
3	252
4	336
5	420
6	504
7	588
8	672
9	756

87	
1	87
2	174
3	261
4	348
5	435
6	522
7	609
8	696
9	783

92	
1	92
2	184
3	276
4	368
5	460
6	552
7	644
8	736
9	828

96	
1	96
2	192
3	288
4	384
5	480
6	576
7	672
8	768
9	864

ting the stencil for the mimeographed sheet of prepared tables it is preferable to use pica type, the larger size of standard typewriter type. The mimeographed sheets should, of course, be discarded as soon as the pupils have learned to make their own tables.

Making the Table of Multiples

It seems desirable that the pupils understand the mathematical principles involved in the construction of a table of multiples by the short method rather than learn this procedure by rote. To achieve this end the teacher will probably find it helpful first to construct a table of multiples by multiplication and then to present the short method. In developing the short method the pupil should be led to see that, while seven times a number can be obtained by adding two times and five times the number, it is preferable to obtain this value by adding three times and four times the number since these multiples are adjacent in the table. When developing the short method of making the table with superior pupils, the teacher may need to use a more difficult divisor, such as 37, since these pupils may not see the value of the short method for a simple divisor, such as 21.

Several teachers suggested that the experimental method would be more satisfactory if the table of multiples could be easily checked for accuracy before it was used. To meet this need the check of the table by adding the last four multiples was devised. Later experience with this check in the final part of this investigation seems to justify the recommendation that pupils always be required to find the sum of the last four multiples and, if this sum is not ten times the third multiple, to correct the table. When the table fails to check, one way to find the error in it is to go back over its original construction. This is the plan that was used by the fifth grade pupils in the comparative study reported in Chapter III. In doing this the error may, of course, be repeated. With the knowledge of the presence of an error, however, more care is likely to be taken in finding the multiples. It would be interesting to experiment with a plan for finding an error in which the check of each item is independent of the original construction. For example, if the sum of the first four items in the table is ten times the divisor, it seems reasonable to assume that the error is not in this part of the table. If no error is found here the fifth multiple might be checked by doubling it and noting if the result equals ten times the divisor. Each remaining multiple could be checked by the addition of the divisor to the multiple which precedes it.

When making a table of multiples the pupil should first draw a vertical line two to three inches long. He should place the number 1 on the left of this line and the given divisor on the right, and then draw a horizontal line directly under these two numbers. After the number 2 and twice the given divisor have been placed under this line, another horizontal line should be drawn. This process of drawing a horizontal line under each calculated multiple should be continued until the completed table, including the sum of the last four multiples, appears as shown at the left. It is suggested that all of the horizontal lines be drawn freehand. If the pupils are permitted to use a ruler for drawing these lines, they usually will waste time. The vertical line may be drawn with a ruler if necessary, but many pupils will probably be able to draw it satisfactorily freehand. It is necessary for the teacher to watch for and to eliminate the tendency of some pupils to take the time to draw unnecessary lines when making the table. The given divisor should be placed far enough to the right of the vertical line to allow room for an extra digit in the larger multiples and for still another digit in the check number without disturbing the alignment of the vertical columns in the table. Some teachers may want to have the pupils indicate by a check mark that the sum of the last four multiples has been compared with the third multiple. The preceding arrangement of the table of multiples seems to promote accuracy in its construction and use. When pupils neglected to draw the horizontal lines, they often failed, after selecting the correct multiple from the

1	37
2	74
3	111
4	148
5	185
6	222
7	259
8	296
9	333
	1110

table, to use for the quotient figure the correct corresponding figure from the left-hand column. Some pupils omitted the left-hand column of the table and as a result frequently used the correct multiple but the wrong figure in the quotient.

It is desirable that the pupils learn to make tables with as much speed as is consistent with accurate and careful work. Not only does the rapid and efficient construction of the table make the solution of a division example more pleasant, but it also tends to prevent errors due to loss of place in the table. To encourage the efficient construction of the tables, the teacher will find it helpful to time the pupils during some of their practice work in making them.

In performing long division by the experimental method, the table of multiples of the divisor should usually be placed on the immediate left of the example and sufficient room for this should be provided on all test papers. However, if the table is to be used for solving a group of division examples having the same divisor, it may be placed on a separate sheet of paper. The table should never be placed on the reverse side of the paper containing the division example; such a position is awkward and is certain to cause errors in copying the numbers from the table.

It is necessary that the teacher locate and correct deficiencies in addition and multiplication by 2 if the instruction in the making of tables of multiples is to be effective. Some pupils may require considerable remedial instruction. It was the opinion, however, of one teacher that one advantage of the experimental method is the ease with which pupil difficulties in long division can be located. In some groups it was found that pupils frequently forgot the carry number when adding two multiples. When this occurs the teacher should check to see that the pupils add the carry number immediately to one of the two digits in the next column. If they wait to add the carry number to the sum of these two digits they are likely to forget it. If the pupils have difficulty in adding the correct two multiples when calculating a larger multiple, it is recommended that they place a finger of the left hand at the side of the two multiples to be added. When checking the table, if the pupils have difficulty in adding the last four multiples, an additional line can be drawn between the fifth and sixth multiples, or the pupil can hold the fingers of his left hand over the first five multiples.

Using the Table in Division

When teaching the pupil how to use a table of multiples of the divisor in long division, the example should be written in the usual way and the division performed as follows:

(1) Select the partial dividend.
(2) Look at the table of multiples and select the multiple that is nearest to but *not larger* than the partial dividend. Write this multiple under the partial dividend.
(3) The figure in the table at the left of the multiple selected in step (2) is the quotient figure. Write this quotient figure over the last figure of the partial dividend.
(4) Subtract.
(5) After subtracting, and before bringing down the next figure of the dividend, compare the remainder with the divisor to see that the remainder is *smaller* than the divisor.
(6) Bring down the next figure of the dividend, and repeat the steps above to find the next quotient figure.

It seems wise to caution pupils that the multiple selected in the table must be smaller than, or equal to, the partial dividend but *also nearest* the partial dividend.

In the initial instruction in the use of the table in long division it is desirable to use only those division examples that have no remainders and no zeroes in the quotients, thus postponing these difficulties until the pupil has learned the pattern of the experimental method. When division examples with remainders are introduced, the teacher will probably find it helpful to have the remainder written as in the following example:

$$\begin{array}{r} 2398\ \text{R}23 \\ 26\overline{)62371} \end{array}$$

After pupils have learned the experimental method of long division they are soon ready to solve division examples of any difficulty. The teacher will find that most of the examples in long division in fifth grade arithmetic textbooks are not sufficiently difficult to challenge the pupils for any length of time. To maintain interest and to take full advantage of the experimental method, it is recommended that the teacher introduce at an early date division examples having more than two figures in the divisor as well as those having more than three figures in the quotient. The pupils will also find it interesting to solve, using one table of multiples, a group of division examples having the same divisor.

It is necessary when teaching the division process to watch for errors in subtraction and to give needed remedial instruction. Some pupils may have difficulty in selecting the correct multiple from the table because they do not understand clearly how to compare the size of numbers. These pupils will need help in developing number concepts.

Checking Long Division

In the preliminary study of the experimental method no fixed or uniform requirement for checking the final quotient was made for the classroom work in long division or for the long division tests. Each teacher was left free to follow her customary procedure in this matter. Most of the teachers required that all work in long division be checked by multiplying the quotient and the divisor and adding the remainder to this product. This process was found to be very time-consuming. As previously mentioned on page 19, Test II was repeated in Group 5A without checking; as a result the mean time for this test dropped from 65 minutes to 37 minutes, or almost half, with little change in accuracy. For some pupils the check by multiplication seems more difficult than the experimental method of long division inasmuch as it requires greater proficiency in multiplication. These pupils found checking confusing and discouraging. When a division example did not check they would be in doubt whether the error was in the division or in the work of the check. Consequently it was difficult to locate the error. Some teachers were of the opinion that the checking of long division, regardless of the method of long division, presents a real problem, and that a check of long division more satisfactory than multiplication is needed. Therefore the author decided to use the check of casting out 9's throughout the experiment which formed the final part of this study. The experience thus gained with the method of casting out 9's indicated that it can be used very successfully in grade five for checking long division.

Little attention is given the method of casting out 9's in modern arithmetic textbooks for the elementary school. This method will be illustrated by checking the long division example shown below. The first step in this method of checking is to "cast out" the 9's (that is, to subtract all the 9's) from the divisor, the dividend, the quotient, and the remainder. A short way to cast out the 9's from the dividend, 181345, is to add its digits, which gives 1 + 8 + 1 + 3 + 4 + 5, or 22, and then to add the digits of 22, which gives 4. This number 4 is called the *check number* of 181345. It is the number remaining after all possible 9's have been subtracted from 181345. It is also the remainder when 181345 is divided by 9. In obtaining the check number by the short method of adding the digits, the process is always continued until a number equal to or less than 9 is obtained. When 9 is obtained, this 9 is cast out by subtraction, which gives 0 as the check number.

```
           ⑧        ③

  ⑧
          6974   R21              check
     26)181345
         156          ④↖            8
          253                        8
          234                       64
           194                       3
           182                      67
            125                ↘
            104                     ④
             21
```

In like manner, the check number of 26 is 8, the check number of 6974 is 8, and that of 21 is 3. These check numbers are given above within circles. To check the division, the check number 8 of the quotient is multiplied by the check number 8 of the divisor, and to the product 64 is added the check number 3 of the remainder. Finally, the check number of the result 67 is obtained and found to be 4. This check number is compared with the check number of the dividend, which is also 4. Since these two check numbers are the same, the work is considered correct. If they were different, there would be an error in the work.

There are some simplifications that can be made in this procedure. In finding a check number by adding the digits of a number, any 9 or group of digits totaling 9 may be omitted without affecting the required check number. For example, when finding the check number of 181345 the digits 1 and 8, and 4 and 5 may be skipped. The check number is easily found by adding the remaining digits 1 and 3.

When using the check of casting out 9's in the elementary school, it helps avoid confusion if the pupil draws a circle around each check number as shown in the preceding illustration. Some teachers like to have the pupil draw the line between the two final check numbers at the end of the work in order to indicate that these numbers have been compared. It may be necessary, particularly in the beginning, for some elementary school pupils to write all the intermediate steps when finding a check number by the short method of adding digits. That is, in finding the check number of 467 the pupil would first write 17, and then write 8, the required check number.

The teacher should, of course, be familiar with those situations in which the check of 9's will not detect an error. When the check number of the divisor is 0, the product of the check numbers of the divisor and the quotient will be 0 regardless of the check number of the quotient. Hence, in this situation, an error cannot be detected by this check unless there is a remainder. It is, however, unlikely that the remainder will be correct when the quotient is incorrect. Therefore, even though the check number of the divisor is 0, an error will usually be detected if the example has a remainder.

If two digits in the quotient or the remainder are transposed, the check of 9's will not detect the error, since this transposition does not affect the check number. For example, the check number of 467 is 8, which is the same as that of 476. However, this type of error is more likely to occur when copying numbers than in the actual process of division.

For further discussion and proofs of the method of checking by casting out 9's the reader is referred to publications by Upton,[2] Davis,[3] and Bond.[4]

It is interesting to note that the check of division by multiplication can be simplified if the experimental method has been used to obtain the result. Since the table of multiples of the divisor is available and has been checked, it may be used in the multiplication of the divisor and the quotient. To check in this way the division example on page 46, the product of the divisor, 26, and the quotient, 6974, is found as shown on the left by taking the required multiples of 26 from the table. After adding the remainder, the result is compared with the dividend. This procedure for checking does not seem as satisfactory as casting out 9's because it requires more time and care. It is given here because of its interest and to show that the table of multiples may also be used effectively in multiplication.

1	26
2	52
3	78
4	104
5	130
6	156
7	182
8	208
9	234
	780

check

$$\begin{array}{r} 26 \\ 6974 \\ \hline 104 \\ 182 \\ 234 \\ 156 \\ \hline 181324 \\ 21 \\ \hline 181345 \end{array}$$

Regardless of the method of checking used, it is believed by the author that emphasis on checking can be profitably postponed until the pupils have achieved an understanding of long division. In this way they will be free at the beginning to concentrate attention on learning a new process.

[2] C. B. Upton, "Professionalized Subject Matter in Arithmetic for Normal Schools," *Teachers College Record,* Vol. XXVII, No. 3 (November, 1925) and No. 5 (January, 1926).

[3] H. T. Davis, *General Mathematics,* p. 16. 1935.

[4] E. A. Bond, *Arithmetic for Teacher-Training Institutions,* pp. 100, 126, 162. 1934.

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CHAPTER III

COMPARATIVE STUDY OF THE EXPERIMENTAL AND USUAL METHODS OF LONG DIVISION

IN THE preliminary study reported in Chapter II, the reactions of the teachers and pupils to the experimental method of long division were sufficiently favorable to justify further consideration of it. In the comparative study reported in this chapter an attempt was made to answer these major questions:

1. If the experimental method of long division is taught to fifth grade pupils, how will the accuracy of their performance when solving a set of long division examples compare, at various stages of instruction, with that of fifth grade pupils taught the usual method?
2. How will the time required by pupils using the experimental method compare with that needed for the usual method?
3. Do the answers to (1) and (2) justify the author in recommending the experimental method?

During the school year 1946-47 one fifth grade class in each of the three elementary schools of the Teachers College of Connecticut participated in this comparative study. In planning the instructional program for the experimental method, use was made of the recommendations, reported in the preceding chapter, which had developed from the initial experience in teaching it to elementary school children. Great care was also taken to make the presentation of the usual method as effective as possible.

Immediately after the beginning of school in the fall the teachers of these three fifth grade classes began a review program in arithmetic, after which, in the first week of October, the pupils were given a test in arithmetic computation comprising 79 examples. The first 23 items of this test covered the simple material in addition, subtraction, multiplication, and short division that seems to be a necessary and desirable background for long division. The remainder of the test was made up of repetitions of this kind of material, with a few more difficult examples at the end. The pupils were allowed 20 minutes for this test. A pupil's score was considered to be the number of correct items; hence 79 was the maximum score possible.

Most of the pupils in these fifth grade classes had received no instruction in long division in the fourth grade and no mention of long division was made during the review period. However, they were tested in long division at this time in order to be sure of their status in regard to this topic. The test used for this purpose was the one used at other times of the year to measure the outcomes of the instruction; this test will be discussed in detail at a later point in this chapter.

The intelligence quotients of the pupils were obtained from the School Department of the City of New Britain. During the fall, as a part of the testing program conducted by this department, all fifth grade teachers in the city administered in their classes the Otis Quick-Scoring Mental Ability Tests.

It was decided to teach the experimental method in two of these fifth grade classes and the usual method in the third class. The amount of time devoted to long division throughout the year was held constant in all groups. Instruction in long division began the week of October 7 and continued three hours a week during that month and the month of November. In the months of December, January, and February two hours a week were devoted to long division, and during the remainder of the year one hour a week was allotted to it. In the typical fifth grade curriculum the study of long division is limited to 2-figure divisors and 3-figure quotients. For experimental purposes the fifth grade program in long division in these three classes was altered to include considerable work with examples having 2-figure divisors and 4-figure quotients, and, toward the end of the year, to include instruction in the solution of examples having 3-figure divisors. Approximately half of the time used for long division after the middle of March was devoted to examples having 3-figure divisors. Uniform practice material for these harder examples was provided by the author for all three classes. All three groups were taught the method of casting out 9's for checking the final quotient. This was the only method of checking used.

Throughout the year the author kept in constant touch with the long division program in each class. He made weekly visits to each class during the first half of the year and frequent ones thereafter, had many conferences with the teachers, and assisted with the administration of the tests. Although student teachers were present at certain times in these classes, all of the teaching of long division was done by the regular teacher of the class.

EXPERIMENTAL GROUP I

Experimental Group I consisted of 25 pupils in a fifth grade class of 30 children. One pupil who was repeating the fifth grade and four pupils who were members of the class for only part of the year were not considered in this group. The intelligence quotients of the group ranged from 81 to 125 with a mean of 99.7. The scores on the test in arithmetic computation, described on page 50, ranged from 21 to 70 with a mean of 43.3. Every child in this group made a score of zero on the long division test given before instruction began, and it was clear from their papers that they had no knowledge of this topic.

The experimental method of long division was introduced in this group by showing the pupils how to use the mimeographed page of tables of multiples in working examples in long division. After about two weeks of such instruction, the pupils were then taught to make their own tables. The check of the table by adding the last four multiples was considered a regular part of the construction of the table and was required at all times. The check of the final quotient was made by casting out 9's but checking was not always required of the pupils.

The teacher of this group had had considerable experience with teaching the experimental method. She had participated in the preliminary study in grade five and had also taught the experimental method at other times.

EXPERIMENTAL GROUP II

Experimental Group II consisted of 27 pupils in a fifth grade class of 31 children. Three pupils in this class were not used because the teacher had planned to give them fourth grade arithmetic, and one pupil was not used because of excessive absences from school. The intelligence quotients of the pupils in this group ranged from 72 to 124 with a mean of 100.6. The scores of this group on the test in arithmetic computation ranged from 31 to 75 with a mean of 47.1. Some of the pupils in this group had received instruction in long division in grade four. On the 12-example test in long division, which preceded the instruction, there was one score each of 10, 7, 3, and 2, and three scores of 1. The remaining twenty papers showed little or no knowledge of long division.

In this group the experimental method of long division was presented by teaching simultaneously the construction of the table of multiples of the divisor and the use of the table in long division. As in Experimental Group I the check of the table was considered an integral part of the process and was required at all times. The final quotient was checked by casting out 9's but checking was not always required.

The teacher of this group was experienced in teaching the experimental method of long division but not to the extent of the teacher of Experimental Group I. She had participated in the preliminary study in grade four instead of in grade five, but had taught the experimental method at other times in grade five.

CONTROL GROUP

The control group consisted of 28 pupils from a fifth grade class of 29 children. The statistical analysis of the test results required the discard of one pupil in this class, as will be explained on page 58. The intelligence quotients of this group ranged from 84 to 132 with a mean of 110.5. The scores on the test in arithmetic computation ranged from 19 to 78 with a mean of

50.7. The control group ranked higher than the two experimental groups with respect to both intelligence quotients and scores on the test in arithmetic computation. The preliminary test in long division indicated that only three pupils of this group had any knowledge of long division. These three pupils made scores of 8, 5, and 2.

It was decided to use in the control group the form of the usual method of long division in which the first figure of the divisor is used as a trial divisor if the second figure is 5 or less, and the first figure of the divisor increased by one is used as a trial divisor if the second figure is greater than 5. The pupils were taught to use inspection to obtain the quotient figure 0; inspection was also used in certain easy situations to obtain the quotient figure 1. For divisors 13, 14, and 15, which were not presented in the early part of the instruction, the pupils were directed to use trial and error instead of the rules given above. This form of the usual method gives the pupil, in the long run, the correct quotient figure on the first trial about four times out of five; and, furthermore, when an incorrect quotient figure is obtained, more than one correction of this figure is seldom needed.

This group was taught by two thoroughly experienced teachers. The first served from the beginning of the year until the second week of December, at which time she was transferred by the college administration from her elementary school assignment to one on the college faculty. The second teacher worked with the group for the remainder of the school year.

CONSTRUCTION AND ADMINISTRATION OF TESTS

To measure the achievement of the pupils in long division during the course of this experiment two tests were constructed.

The first test, called Test A, consisted of 12 division examples having 2-figure divisors and 4-figure quotients; this test is shown on page 54. Half of the divisors of the test examples end in a figure less than 5 and half end in a figure greater than 5. There are no repetitions of figures in any 4-figure quotient. When these 12 examples are solved by the usual method, as taught in the control group, the correct quotient figure is obtained on the first trial 30 times, the quotient figure must be corrected once 16 times, and it must be corrected twice 2 times. Thus this test measures the ability of the pupils in the control group to correct quotient figures, and at the same time it does not give the pupils in the experimental groups an advantage by containing an unreasonable number of situations in which the quotient figure must be corrected.

It seemed desirable to give this test in two parts, with an intermission of at least 20 minutes between them, in order to avoid asking the pupils to work steadily with the same kind of material for too long a period of time. A time limit of 30 minutes was set for each part of the test in order to allow almost all pupils to solve the six test examples in each part. The pupils were directed to work as rapidly as possible without making mistakes. They were also told to correct any errors they might discover while solving an example but not to go back over the work when an example was completed and not to check the final quotient by any method. The pupils in the experimental groups were required to construct the table of multiples at the left of each example on the test paper and to check, and correct if necessary, this table before using it. Sufficient room for the table was provided on all test papers. The pupils were directed to take their test papers to the teacher or to her assistant as soon as they were through work. Then the teacher or her assistant recorded, to the nearest minute, the time when each pupil finished each part

TEST A, PART I

$42\overline{)208194}$ $64\overline{)531615}$ $56\overline{)164997}$

$23\overline{)160425}$ $37\overline{)313372}$ $78\overline{)637531}$

TEST A, PART II

$47\overline{)180791}$ $63\overline{)428763}$ $77\overline{)553218}$

$26\overline{)221936}$ $34\overline{)270164}$ $84\overline{)581917}$

of the test. If any pupils were still working at the end of 30 minutes they were stopped at that time. A pupil's score on this test was considered to be the number of examples with correct answers. In addition to this score, the total time used by each pupil was computed.

During the week of December 9 Test A was given for the first time after instruction in long division began. The pupils in the experimental groups could have been tested earlier than this, for after a short period of instruction most of them seemed to understand the experimental method and to need only the development of facility in its use. However, no testing was done until the several different techniques of the form of the usual method of long division used in this study had been presented to the control group. After a three-day interval without instruction, Test A was repeated for the purpose of studying the reliability of this test. During the week of February 17 the pupils were again given Test A. After the elementary schools' vacation period of the week of February 24, the pupils were tested again to see if a break in practice had materially affected their skill. During the last week of May, Test A was given for the final testing of the pupils' achievement with examples having 2-figure divisors.

The coefficient of correlation for the two sets of scores on Test A obtained in December is .75. For each testing in December, the coefficient of correlation for the scores obtained on the two parts of Test A was also computed. These coefficients of correlation are equal to .58 and .59; when the Spearman-Brown formula is applied, the estimates of the reliability of Test A are .73 and .74. Thus the reliability of Test A seems to be sufficiently high for making group comparisons but not for making individual predictions. If familiarity with the test examples had played an important role when Test A was repeated, the correlation coefficient obtained from the two testings in December would presumably have been much higher than the coefficients obtained by correlating halves of the test and applying the Spearman-Brown formula. The fact that the retesting coefficient was only .75 while the other values of r were .73 and .74 seems to imply that memory had not created a spurious appearance of reliability. It is interesting that the pupils did not seem to be conscious at any time during the year that the same test was being repeated.

The second test, Test B, consisted of 6 division examples having 3-figure divisors and 4-figure or 5-figure quotients; it is shown on this page. Half of the test examples have a divisor with a second figure less than 5 and half have a divisor with a second figure greater than 5. As in Test A, the number of corrections of the trial quotient figures needed when the examples are solved by the usual method, as taught in the control group, was restricted to a reasonable number. The correct quotient figure is obtained on the first trial 16 times, the quotient figure must be corrected once 10 times, and it must be corrected twice 2 times. Since examples of this difficulty are usually not taught in grade five and since no attempt was made to complete the teaching of this kind of material, it seemed to the author that the length of this test should be limited to 6 examples. The test was administered in two parts, with a time limit of 30 minutes for each part and with an intermission of 20 minutes or more between the two parts. The directions and the method of scoring were exactly the same as those used for Test A.

TEST B, PART I

$328\overline{)971972}$ $172\overline{)6755384}$ $623\overline{)45445329}$

TEST B, PART II

$782\overline{)6640795}$ $237\overline{)5917605}$ $469\overline{)16781784}$

In March, before instruction involving 3-figure divisors, the pupils were given Test B to determine whether or not their skill with 2-figure divisors would transfer without practice to this new situation. When giving the test directions to the control group, the author told the pupils that they were being tested on examples that had not been included in their lessons and pointed out how the principles they had already learned in regard to trial divisors applied to 3-figure divisors. Similarly, it was pointed out to the experimental groups at the time directions were given them that a table of multiples of a 3-figure divisor could be made and used in exactly the same manner in which they had made and used the table of multiples of 2-figure divisors. However, no practice with 3-figure divisors was given in any of the three groups before the first administration of Test B.

Test B was given again during the first week of June after the pupils had been taught to solve division examples having 3-figure divisors. The coefficient of correlation for the scores on the two parts of Test B obtained at this time is .39. The Spearman-Brown formula gives, for this value of *r*, an estimate of .56 for the reliability of Test B.

STATISTICAL METHOD

The achievement of the pupils in each experimental group was compared with that of the pupils in the control group by a technique[5] which makes it unnecessary to reject more than a few, if any, cases to secure matching and which, at the same time, takes into account background traits in making a comparison. In this study it was decided that two background traits, intelligence quotient and ability in arithmetic computation, were likely to affect seriously achievement in long division. The pupils in each group were therefore divided into the following three subgroups with respect to intelligence quotient: below 99, 99–115, above 115. The pupils in each of these subgroups were again divided into three subgroups with respect to score on the test on arithmetic computation as follows: below 40, 40–57, above 57. By this procedure the pupils in each experimental group and in the control group were classified into subgroups or categories relatively homogeneous with respect to intelligence quotient and to ability in arithmetic computation as measured by the arithmetic test given in October. This classification might have produced nine subgroups. However, there were only seven subgroups, for in the lowest intelligence quotient range there were no pupils with scores above 57 on the arithmetic test and in the highest intelligence quotient range there was only one pupil with an arithmetic test score below 40. This one pupil was the only case that had to be discarded for purposes of statistical analysis. The number of pupils in each category is shown in Table 7. A chart was made giving the names of the pupils in each category so that scores could be properly classified.

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[5] Palmer O. Johnson and Jerzy Neyman, "Tests of Certain Linear Hypotheses and Their Application to Some Educational Problems," *Statistical Research Memoirs,* Vol. I, 1936, p. 57.

TABLE 7

NUMBER OF PUPILS IN EACH CATEGORY OF THE EXPERIMENTAL GROUPS AND THE CONTROL GROUP

	C1	C2	C3	C4	C5	C6	C7
	Intelligence Quotient						
Group	*Below 99*		*99–115*			*Above 115*	
	Arithmetic Test Score						
	Below 40	*40–57*	*Below 40*	*40–57*	*Above 57*	*40–57*	*Above 57*
Exp. Group I	4	9	5	3	2	1	1
Exp. Group II	7	6	1	6	4	2	1
Control Group	5	1	3	5	3	3	8

In comparing two distributions of scores, a chart like that in Table 8 was made. In this case, shown to illustrate the method, the scores of Experimental Group I are entered in the X-columns and the scores of the control group in the Y-columns. The necessary calculations are indicated at the bottom of this chart.

To test the hypothesis that $\overline{X} - \overline{Y}$ is zero we compute

$$F = \frac{S_b^2\,(N - S - 1)}{S_a^2(R)},$$

where

$$S_b^2 = \frac{\left\{\Sigma\,(\overline{X} - \overline{Y})\,\dfrac{N_xN_y}{N_x + N_y}\right\}^2}{\Sigma\,\dfrac{N_xN_y}{N_x + N_y}}$$

$$S_a^2 = \Sigma\left\{\Sigma X^2 + \Sigma Y^2 - \frac{(\Sigma X + \Sigma Y)^2}{N_x + N_y}\right\} - S_b^2.$$

N = total number of cases S = number of categories

R = number of relations tested, which in this case is 1.

TABLE 8

WORK CHART, COMPARISON OF TEST A SCORES, EXPERIMENTAL GROUP I AND CONTROL GROUP, FIRST TESTING

	C1		*C2*		*C3*		*C4*		*C5*		*C6*		*C7*		*Totals*
	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y	
	9 2 7 5	7 6 8 0 10	11 8 9 12 10 12 8 8 9	9	6 0 9 6 5	2 10 4	9 12 12	9 8 6 3 4	12 8	8 8 8	11	9 7 10	11	11 10 10 8 9 11 8 8	
Sum of squares	159	249	863	81	178	120	369	206	208	192	121	230	121	715	3,812
Sum of 1st powers	23	31	87	9	26	16	33	30	20	24	11	26	11	75	
	54		96		42		63		44		37		86		—
Number of scores	4	5	9	1	5	3	3	5	2	3	1	3	1	8	53
Mean	5.75	6.20	9.67	9.00	5.20	5.33	11.00	6.00	10.00	8.00	11.00	8.67	11.00	9.38	—
$\overline{X} - \overline{Y}$	−0.45		0.67		−0.13		5.00		2.00		2.33		1.62		—
$\frac{N_x N_y}{N_x + N_y}$	2.22		0.90		1.88		1.88		1.20		0.75		0.89		9.72
$\frac{(\Sigma X + \Sigma Y)^2}{N_x + N_y}$	324.00		921.60		220.50		496.13		387.20		342.25		821.78		3,513.46
$(\overline{X} - \overline{Y}) \frac{N_x N_y}{N_x + N_y}$	−1.00		0.60		−0.24		9.40		2.40		1.75		1.44		14.35

In the formula for S_b^2 the summations are run over all the categories. In the formula for S_a^2 the summations within the brackets are made for each category, and then the summation indicated outside the brackets is run over all the categories.

Here one estimate of the variance based on variation between groups, $\frac{S_b^2}{1}$, is being compared with another estimate of the variance based on variation within groups $\frac{S_a^2}{N - S - 1}$. The number of degrees of freedom associated with the first estimate is 1 and that with the second is $N - S - 1$. The test of significance is made by referring to the F table.[6] To continue with the illustration:

$$S_b^2 = 21.19 \qquad S_a^2 = 277.35 \qquad F = 3.44$$

In the F table we find for $n_1 = 1$ and $n_2 = 44$ that $F \geqq 4.06$ would be expected in 5 per cent of random samples. Hence in this case, since $F = 3.44$, we do not have sufficient evidence to reject the null hypothesis that $\overline{X} - \overline{Y} = 0$.

In Table 8 the quantity $\Sigma\,(\overline{X} - \overline{Y})\,\frac{N_x N_y}{N_x + N_y}$ is positive, which means that the difference between these two sets of data is in favor of the X-scores, or the experimental group. However, the foregoing analysis shows that this difference can be accounted for as the result of sampling errors. If this quantity were negative, the difference would be in favor of the Y-scores, or the control group.

RESULTS OF COMPARATIVE STUDY RELATING TO ACCURACY

The distributions of the scores obtained at each testing are given, together with their means, in Table 9 and Table 11. The technique previously explained was used to compare the scores of the pupils in each experimental group with the scores of the pupils in the control group at each testing, and to compare the two distributions of scores in the experimental groups at

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[6] George W. Snedecor, *Statistical Methods*, p. 184. 1938.

TABLE 9

Distributions and Means of Test A Scores in All Groups

Score	Experimental Group I					Experimental Group II					Control Group				
	Dec. (*1*)	*Dec.* (2)	*Feb.*	*Mar.*	*May*	*Dec.* (*1*)	*Dec.* (2)	*Feb.*	*Mar.*	*May*	*Dec.* (*1*)	*Dec.* (2)	*Feb.*	*Mar.*	*May*
12	5	4	3	6	5		2	3	4	4		4	3	2	4
11	3	4	8	6	6	4	2	4	7	8	2	3	3	5	7
10	1	2	5	1	3	2	1	6	4	5	5	4	6	5	5
9	5	6	2	3	3	5	7	6	5	6	4	3	7	5	4
8	4	5	3		5	4	3	2	2	2	8	5	4	5	4
7	1		2	1	1		5	1	3	1	2	3	2	2	2
6	2			4	1	3	1	1	1		2	4		3	1
5	2			2		3	2	1					2	1	
4		3	1		1	2	1	1			2		1		1
3				1		2	1				1				
2	1		1	1				1	1		1	1			
1								1				1			
0	1	1				2	2			1	1				
Total	25	25	25	25	25	27	27	27	27	27	28	28	28	28	28
Mean	8.44	8.72	9.44	8.84	9.60	6.96	7.44	8.70	9.44	9.74	7.54	8.43	9.00	8.96	9.54

each testing. For each of these comparisons involving 2-figure divisors Table 10 gives $\Sigma (\overline{X} - \overline{Y}) \frac{N_x N_y}{N_x + N_y}$, the value of *F* obtained, and the value of *F* at the .05 point. The value of *F* at the .01 point is also given in those situations in which it is of interest. Table 12 gives the same information for the comparisons in regard to 3-figure divisors.

Two-Figure Divisors

For 2-figure divisors all but two of the ten differences between the experimental groups and the control group are positive and thus in favor of the pupils in the experimental groups. However, in Table 10 an examination of the values of *F* obtained in each of these comparisons shows that none of these differences is significant. We therefore cannot generalize from these data that pupils solve long division examples of the difficulty of those in Test A more accurately when using the experimental method. It is a tenable conclusion from the data in regard to examples of this difficulty that, in the population from which these pupils can be considered a random sample, there is no difference with respect to accuracy between the two methods at any time in grade five after both methods have been completely presented. It should be noted, however, that the number of pupils involved in these comparisons was small; hence a difference must be large in order to be statistically significant. By referring to the work chart in Table 8 a rather substantial difference will be found, yet it is not significant.

It is possible, of course, that a real difference exists between the two methods and that it was not detected by this experiment because the factor of teacher variability was not taken into account in the experimental design. Teacher differences may have systematically operated in favor of either of the two methods of long division. Only a very limited number of fifth grade classes were available to the author for study in the elementary schools administered by the Teachers College of Connecticut. Under these circumstances it seemed best to undertake a carefully controlled experiment in these schools and to attempt to equalize

TABLE 10

COMPARISONS OF TEST A SCORES

Experimental Group I and Control Group; Experimental Group II and Control Group; Experimental Group I and Experimental Group II

Comparison	$\Sigma(\overline{X}-\overline{Y})\frac{N_xN_y}{N_x+N_y}$	S_b^2	S_a^2	$N-S-1$	F
Experimental Group I (X)					
Control Group (Y)					
December (1)	14.35	21.19	277.35	45	3.44[a]
December (2)	10.78	11.96	265.07	45	2.03
February	6.68	4.59	182.47	45	1.13
March	.82	.07	287.10	45	.01
May	2.75	.78	151.14	45	.23
Experimental Group II (X)					
Control Group (Y)					
December (1)	−1.79	.29	373.99	47	.04[b]
December (2)	−3.01	.82	354.00	47	.11
February	4.08	1.51	228.05	47	.31
March	10.21	9.43	200.90	47	2.21
May	6.30	3.59	205.11	47	.82
Experimental Group I (X)					
Experimental Group II (Y)					
December (1)	20.48	36.54	226.80	44	7.09[c]
December (2)	19.04	31.58	345.72	44	4.02
February	10.86	10.27	214.67	44	2.10
March	−5.59	2.72	295.27	44	.41
May	−1.15	.12	207.58	44	.03

[a] $F_{.05} = 4.06$
[b] $F_{.05} = 4.05$
[c] $F_{.05} = 4.06$ $F_{.01} = 7.24$

the teacher factor as much as possible by using only thoroughly experienced teachers of recognized ability.

One of the reactions of the teachers who have taught the experimental method is the feeling that it can be more easily and quickly taught. It is interesting to note that the value of F obtained in comparing Experimental Group I and the control group at the first testing in December was somewhat near the value of F at the .05 point. The difference obtained at this time, although not significant, is less likely to happen by chance than the other differences obtained. Furthermore, Table 9 shows that at this time there were five perfect papers in Experimental Group I and none in the other groups. As might have been expected the differences between the three groups were small at the end of the year.

When the two experimental groups were compared with respect to achievement with 2-figure divisors, the difference between them was significant at the .05 level at the first testing in December and was in favor of Experimental Group I. At the second testing in December the difference between them was very nearly significant at the .05 level. These differences can be accounted for in two ways. The mimeographed tables of multiples were used in Experimental Group I at the beginning and this procedure seems to create more interest in the topic of long division. On the other hand, these differences may have been due to a technical matter in the instruction of Experimental Group II which escaped the attention of the author until after the December testings. The pupils in this group had worked out many of their practice examples on ruled paper without drawing the small horizontal lines of the table. Their lack of practice in drawing these lines, even though it is a small matter, may have adversely affected the results of the first two tests. Although there is no evidence to this effect, it is the opinion of the author that both factors operated to produce the difference between the two experimental groups in December. The differences between these two groups were not significant at any other time.

Three-Figure Divisors

The pupils took Test B for the first time in March before they had been given any practice in long division with 3-figure divisors. The highest mean score occurred in Experimental Group II. The differences obtained between the scores of the experimental groups and the control group were in favor of the experimental groups; but on analysis it was found, as is seen by the values of F in Table 12, that these differences were not significant. Thus there is no evidence that the experimental method transfers more successfully to a new situation than the usual method.

TABLE 11

Distributions and Means of Test B Scores in All Groups

Score	*Experimental Group I*		*Experimental Group II*		*Control Group*	
	March	*June*	*March*	*June*	*March*	*June*
6		6	2	3	2	1
5	2	4	2	6	1	2
4	5	4	3	6	2	10
3	4	5	3	3	7	5
2	2	5	6	3	5	6
1	7	1	9	5	6	3
0	5		2	1	5	1
Total	25	25	27	27	28	28
Mean	2.12	3.92	2.37	3.41	2.21	3.07

The two experimental groups were again compared with the control group at the end of the year after instruction in the solution of examples having 3-figure divisors. In the comparison of Experimental Group I and the control group the value of *F* obtained is 4.58, which is between the values of *F* at the .05 and .01 points. It is interesting to note in Table 11 that six pupils wrote perfect test papers in Experimental Group I as compared with three in Experimental Group II and one in the control group. In the comparison of Experimental Group II and the control group the value of *F* obtained is 1.15, which is less than the value of *F* at the .05 point. Although no attempt was made to do a thorough job of teaching long division with 3-figure divisors, the data indicate the possibility that the experimental method, as taught in Experimental Group I, gives more accurate results than the usual method when pupils use it to solve division examples of the difficulty of those in Test B. Thus the experimental method may be the preferable method for examples of this difficulty.

The two experimental groups did not differ significantly with respect to Test B scores in March or in June.

If the experimental method of long division appeals to a teacher, she can use it with the knowledge that the accuracy of the results obtained for examples having 2-figure divisors and

TABLE 12

Comparisons of Test B Scores

Experimental Group I and Control Group; Experimental Group II and Control Group; Experimental Group I and Experimental Group II

Comparison	$\Sigma(\overline{X}-\overline{Y})\frac{N_xN_y}{N_x+N_y}$	S_b^2	S_a^2	$N-S-1$	F
Experimental Group I (X) Control Group (Y)					
March	6.37	4.17	93.78	45	2.00[a]
June	9.42	9.13	89.76	45	4.58
Experimental Group II (X) Control Group (Y)					
March	7.10	4.56	116.49	47	1.84[b]
June	4.82	2.10	85.72	47	1.15
Experimental Group I (X) Experimental Group II (Y)					
March	−3.71	1.20	125.79	44	.42[c]
June	3.94	1.35	81.51	44	.73

[a] $F_{.05} = 4.06$ $F_{.01} = 7.24$
[b] $F_{.05} = 4.05$
[c] $F_{.05} = 4.06$

4-figure quotients has not been shown to differ significantly from that obtained with the form of the usual method taught in this experiment. Furthermore, there is a presumption that more accurate results with the experimental method may be obtained for examples with 3-figure divisors of the difficulty of those in Test B. In order to achieve best results with the experimental method in the early period of instruction it may be preferable to use the teaching plan which begins instruction with the mimeographed tables of multiples.

Results Relating to Time

For the non-professional use of the average person it seems to the author that, within reasonable limits, the speed of a method of long division is of minor importance. When a large amount of division is to be done and speed is important, the work is usually done on a computing machine. For occasional use, accuracy and absence of strain seem to be the important characteristics of a method of long division. Nevertheless, if the experimental method of long division is to be given serious consideration it is necessary to know, at least approximately, how much more time it will be expected to require than the usual method.

To obtain a comparison of the time required for the use of the experimental method and the usual method, the time correct to the nearest minute when each pupil handed in his test paper was recorded on the paper by the teacher or her assistant. Thus it was possible to calculate the time used by each pupil for the tests. The mean time of each group at each testing is shown in Table 13.

It is seen in Table 13 that in each group the mean time used for Test A decreased throughout the year. For examples of the difficulty of those in Test A these data show that the fifth grade pupils needed more time when using the experimental method than they needed when using the usual method as taught in the control group. In May the mean time in Experimental Group I was 34 per cent greater than the mean time in the control group, and in Experimental Group II the mean time was 49 per cent greater than in the control group. Part of this difference in time was undoubtedly due to the fact that the intelligence quotients and the arithmetic test scores were higher in the control group than in either experimental group. However, the greater part of the difference was probably due to the time it takes to construct the table of multiples, and to check and correct it. Fifth grade children, if permitted, may waste considerable time in making the table by using a ruler to draw all the lines needed in the table and to draw unnecessary lines. Hence, in order to keep the time needed for the construction of the table at a minimum the teacher must give careful attention to the mechanical details of making the table.

It should be noted that the time required to solve a long division example by the usual method depends to some extent upon the number of corrections that must be made to the trial quotient figures. If Test A is worked by the form of the usual method taught to the control group, it is necessary, as stated on page 54, to correct the trial quotient figure once 16 times and twice 2

TABLE 13

MEAN TIME IN MINUTES USED BY EACH GROUP FOR TEST A AND TEST B

	Experimental Group I	*Experimental Group II*	*Control Group*
TEST A			
December (1)	48.4	54.6	38.5
December (2)	44.2	49.2	34.8
February	40.1	46.8	32.2
March	38.8	41.3	32.1
May	37.2	41.2	27.7
TEST B			
March	35.2	43.9	37.9
June	31.8	35.5	27.2

times. If Test A had been so constructed as to require more corrections of trial quotient figures, it seems reasonable that the control group would have needed more time for this test. On the other hand, it is doubtful that such a change in Test A would have affected the time used by the experimental groups.

The differences in the mean times of the three groups were much smaller for Test B than for Test A, as can be seen in Table 13. When this test was first given the shortest mean time occurred in Experimental Group I. After all groups had received instruction in long division with 3-figure divisors the time needed for Test B in June had decreased in all groups and was least in the control group. The mean time in the experimental groups was 17 per cent and 31 per cent greater than the mean time in the control group. For the solution of examples of the difficulty of those in Test B the additional time needed when the experimental method is used does not seem to be of great consequence.

METHODS OF CHECKING

The preliminary experiment showed very clearly that checking the final quotient in long division, for the reasons given on page 45, is a serious problem regardless of the method of long division. It also showed that a check of the table of multiples used in the experimental method of long division is very desirable. The latter problem was solved by devising the check of the table previously explained and incorporating it into the experimental method as a regular part of the process. The two teachers of the experimental groups approved of this check of the table, and the pupils in these groups used it very successfully. The experiences with this check during the comparative study indicate that it was a wise addition to the experimental method.

The method of checking the final quotient by casting out 9's was tried out in all three groups that took part in this experiment. This was the only method used for checking the final quotient. In each of the three groups the children reacted very favorably to this method of checking. They learned the method easily and used it successfully. All four of the teachers who participated in the comparative study said that they liked it and thought it simplified the problem of checking long division. Suggestions relating to teaching this check are given at the end of Chapter II.

CHAPTER IV

SUMMARY AND IMPLICATIONS

SUMMARY

THIS INVESTIGATION, consisting of two parts, was undertaken for the purpose of obtaining information and data bearing on the possible values of the experimental method of long division, described on page 5, in which use is made of a table of multiples of the divisor.

In the first part of the study, which was informal and exploratory in character, an attempt was made to find out whether or not teacher and pupil reaction would be sufficiently favorable to this experimental method of long division to justify its more formal study, and to secure suggestions on how to teach it effectively to elementary school pupils. In this preliminary study the experimental method was taught in ten elementary school classes in grades four, five, and six. Chapter II gives a detailed report of the experiences in these classes.

The second part of the investigation, which is reported in Chapter III, was a comparative study of the experimental method of long division and the usual method as described on page 53. The experimental method was taught to two groups of fifth grade pupils and the usual method to one group of fifth grade pupils. The accuracy of the performance, when solving long division examples of certain degrees of difficulty, of the pupils taught the experimental method was compared, by the statistical technique explained on page 57, at various stages of instruction with that of the pupils taught the usual method. For making these comparisons the pupils were divided into categories relatively homogeneous with respect to intelligence quotient and ability in arithmetic computation. A comparison was also made of the time required by the pupils to use the two methods of long division.

The major findings of this study may be summarized as follows:

1. The differences between the experimental method of long division and the usual method with respect to the accuracy of the performance of pupils when solving examples having 2-figure divisors and 4-figure quotients were not statistically significant at any time in grade five after both methods had been completely presented.

2. When the experimental and usual methods were compared at the end of grade five in regard to accuracy with examples having 3-figure divisors and quotients of 4 and 5 figures, one difference was statistically significant at the .05 level and in favor of the experimental method; the other difference was also in favor of the experimental method but was not significant.

3. It was found that the pupils in grade five required more time to complete the test consisting of examples having 2-figure divisors and 4-figure quotients when they used the experimental method than when they used that form of the usual method taught in this experiment. At the end of grade five the mean time for this test in one experimental group was 34 per cent greater than the mean time in the control group, and the mean time in the other experimental group was 49 per cent greater. In interpreting these percentages it must be remembered that the pupils using the experimental method checked and corrected, if necessary, the table of multiples and that the control group was superior to the experimental groups in both intelligence quotient and ability in arithmetic computation.

4. When the test which consisted of examples having 3-figure divisors and quotients of 4 and 5 figures was given at the end of grade five, the mean time in one experimental group was 17 per cent greater than the mean time in the control group, and the mean time in the other experimental group was 31 per cent greater. The smallest mean time occurred, however, in one of the experimental groups when this test was first given before instruction involving 3-figure divisors began.

5. The reactions of nine of the ten teachers who taught the experimental method of long division in the preliminary study were favorable to it. The opinions about it most frequently expressed by these teachers were that it is easier for pupils to understand than the usual method, that it can be more quickly and easily taught, and that it is very helpful to the slow pupil.

6. The method of checking the final quotient in long division by multiplying the quotient by the divisor and adding the remainder was found, in the preliminary study, to be time-consuming and unsatisfactory for many pupils. The teachers who participated in the fifth grade comparative study used only the method of casting out 9's for the check of the final quotient. These teachers were of the opinion that this method of checking long division is much more satisfactory than checking by multiplication, and they recommend its use.

For the detailed suggestions that were developed for teaching the experimental method to elementary school children the reader is referred to Chapter II.

IMPLICATIONS

The findings and the experiences of this study suggest the following uses of the experimental method of long division in the elementary school:

1. The experimental method has a place in the introductory work in long division for the purpose of giving emphasis to the meaning of the long division process. It is rather generally recognized today that one of the principal shortcomings of the elementary school program in arithmetic is the lack of understanding on the part of the pupil of the processes that he is learning. The experimental method of long division gives the teacher a procedure that seems easily understood by elementary school pupils, and therefore it can be used to advantage in the beginning instruction in long division.

2. The experimental method of long division is somewhat more flexible than the usual method with respect to the year in the elementary school in which it can be taught. If the need for long division arises in school activities, this method can be pre-

sented earlier than the usual method since it does not require such a complete background in multiplication. In the preliminary study it was found that the experimental method can be taught to fourth grade pupils.

3. The experimental method gives the teacher an alternate method of teaching long division. The data obtained in this study do not indicate which of the two methods is preferable for examples having 2-figure divisors. However, for examples of this kind, it seems especially valuable for the experimental method to be available for pupils who have failed to achieve competence in long division with the usual method and are in need of remedial instruction.

4. The experimental method provides the teacher with a method of long division that may give more accurate results with examples having 3-figure divisors and 4 or more figures in the quotient. Moreover, for examples of this difficulty the additional time needed for the experimental method appears to be of minor consequence. It seems that the experimental method is more likely to be advantageous in the more difficult work in long division. It is possible that there is some level of difficulty, not necessarily the same for all pupils, below which the usual method is more desirable and above which the experimental method is to be preferred.

5. The experimental method of long division greatly simplifies the solution of a group of division examples having the same divisor. It should therefore be available to elementary school pupils and to adults for use in such situations.

SUGGESTIONS FOR FURTHER STUDY

1. It would be worth while to make a large-scale study of the experimental method of long division in grade five. In this present study eight of the ten differences between the two methods relating to 2-figure divisors were in favor of the experimental method, but were not statistically significant. Differences of the size of some of these might prove to be significant if obtained in the study of a larger group of pupils. The study of a larger group might make it possible to answer the question of the pref-

erable method of long division in terms of the background traits of the pupil as well as in terms of the difficulty of the division examples. Furthermore, a study of the experimental method is needed in which it is possible to generalize with respect to teachers as well as pupils. One limitation of this study is the impossibility of discriminating between differences due to the teachers and differences due to the methods of long division. No attempt was made to analyze the teacher factor; hence teacher differences may have operated systematically in favor of either method of long division. A large-scale study would, of course, have as a prerequisite the training of a number of teachers for teaching the experimental method.

2. Since the experiences of the preliminary study indicated that the experimental method has good possibilities for being of value in remedial instruction in long division, a comparative study should be made of the results achieved with the two methods in a program of remedial instruction in long division. Such a study would, presumably, include those pupils who had failed to achieve competence in long division when originally taught the usual method, and could profitably be conducted at several different grade levels.

3. Inasmuch as the advantages of the experimental method of long division seem to increase with the difficulty of the example, it would be valuable to make a comparative study of this method and the usual method in grade seven or in grade eight. The comparisons of interest would be those relating to examples having divisors of 3 and 4 figures and quotients of 4 or more figures. For long division of this level of difficulty it is possible that the experimental method has distinct advantages over the usual method.

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